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EVALUATING ATTRACTIVENESS OF THE CENTRAL AND THE EASTERN EUROPEAN COUNTRIES BY USING INDEX APPROACH FOR THE STRATEGIC DECISION MAKING PROCESS RELATED TO EXPANSION OF THE FINANCIAL SERVICE MARKETS

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Abstract. Current paper examines the opportunity of detection of the economic-financialattractiveness / potential of the sixteen countries of the Central and Eastern Europe, excluding Germany (the eastern part of what, German Democratic Republic, did belong to the Eastern European region) and Kosovo by the values of their key social political indicators, macroeconomic performance data, and business environment factors for the period of 2010-2016. The methodology suggested in the current work is based on the apparatus of the theory of inverse and ill-posed problems.

Keywords: *economic-financial attractiveness, export of luxurious services, mathematical model, inverse problem, stable solution.*

AMS Subject Classification: *91B64, 91B82, 65F22, 65F22, 97M40*.

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1. Introduction. In-depth specification of investigated problem

Firstly, one should underline that, in this paper, evaluation of the economic-financial attractiveness / potential of the Central and Eastern European countries is conducted only from the standpoint of opportunity to establish exports of banking and other luxurious services to these countries. After having discussed that with my Economics and Finance focused colleagues from the BA School of Business and Finance (Riga, Latvia), the authors of that idea have come up with a set of 43 annually measured social political, macroeconomic, and financial parameters, which, as for us, play the key role in detection of a country's economic-financial attractiveness (see [76] and given respective references). Evidently, the suggested set of parameters cannot be seen as the "full and absolute" set, which is enough and sufficient for evaluation of the customer segment in the Central and Eastern European countries for exporting banking and other luxurious services there: quite possible that investment specialists as well as such people in the domains of financial services, social political estimations, etc. do not agree with the suggested selection of parameters, willing to suggest their own sets from a bigger amount of indicators (they can be found it databases of The World Bank and other comparable transnational organizations).

Country's economic-financial potential is being defined by the values of different economic, social, ecological, scientific technical, politic, etc. criteria, which usually are mutually related and affected. Each of these criteria consists of non-uniform variety of economic, financial, social, political, legal, educational, scientific innovational, ecological, cultural, etc. factors; moreover, each of these factors can consist either of independent sub-factors, or of dependent sub-factors, which are called "indices", "indicators", or "parameter". For example, as it can be concluded from the listed sources [105] that the factor of the level of attractiveness for doing business in a country consists of the six independent indices; democracy factor also consists of six independent indices; global competitiveness factor consists of one index only (called Global Competitiveness Index); corruption level factor is detected by the range of one to nine indices, where part of them are dependent and part are independent; commercial structure and market relations development factor consists of no less than five indices, depending on the region where a country is located and on the evaluation methods that number can go up to seventeen; depending on the methodology, economic freedom factor can consist of seven to nine indices; global risk acknowledgement andconditions of doing business factor consists of six independent indices; depending on region where a country is located as well as on the methodology applied, ecology and ecological risk acknowledgement level factor consists of no less than 10 independent and dependent indexed; human development factor is defined by three dependent indices – literacy and education index, life expectancy index, and quality of life index, which, generally speaking, due to its importance is often considered as separate factor with many sub-factors being included into it; demographic factor is characterized by one to five independent and dependent indices; economic development level factor can consist of up to twenty factors most of what are dependent, etc.

Country's economic-financial potential should be considered as aggregate characteristics of the level of economic, social, political, and legal development of a country (see [15, 60-62, 85] and respective references given there). If, for example, attention is focused only on the level of countries economic development, that level includes two factors – country's economic resources and its economic performance [85]. Country's economic resources characterized by the total volume, structure, and quality represent all accumulated by it and disposed both inside the country and outside of it material values, scientific, intellectual, informational, and labor resources including also entrepreneurial capabilities and natural resources. Country's economic performance characterized by the total volume, structure, quality, and technical level of manufactured goods and services is made by Gross Domestic Product (GDP), Gross National Product (GNP), National Income (NI) as well as by the physical volumes of manufacturing of selected kinds of goods, which do possess temporary strategic value in given circumstances.

Currently, there are many principles, methods, and algorithms used for evaluation of economic-financial attractiveness / potential of a country. Developed in the framework of the decision making theory (see [9, 11, 12, 26, 31, 35, 36, 77, 96, 100, 109] and respective references given there), statistical analysis (see the fundamental works [45, 82] and respective references given there, as well the articles [8, 54, 58]), expert evaluations (see [4, 7, 13, 18, 19, 24, 29, 30, 32-34, 46, 48, 68, 70-72, 79, 80, 89, 95, 97, 98, 104, 110] and respective references given there), and on the evaluation of the mentioned mathematical disciplines, they have depths of different levels, different adequacies of application and complications. Number of such principles and methods exceeds 1000, therefore, it is nearly impossible to study them fundamentally and conduct comparative analysis while preparing one or two, or even more academic papers. Nevertheless, authors of this paper have conducted detailed analysis of themost popular out of these principles, methods, and algorithms (around 70: for example, method of sum of ranking differences, numerical scoring method, multivariatemeanmethods, "Pattern" method, multidimensional comparative analysis methods, different by level and complexity methods of deterministic and stochastic factor analysis, numerous methods of expert evaluation, etc.).The analysis conducted shows that none of them is in condition to detect which out of the indicators and indices are making sufficient impact in selected period or moment of time (and how influential that impact is?) on the economic-financial potential of a concrete country or a group of countries with more or less identical economic, technological, social, political, etc. conditions. Moreover, by these methods one cannot objectively and unconditionally detect how different / identical are the same indicators or indices in different countries, where values of the economic and financial potentials differ each from another dramatically or, alternatively, are very close. In other words, the analysis conducted by authors has shown that widely applied methods do not let objectively detect each economic, financial, social, etc. index / indicator's impact on the economic-financial attractiveness both in a moment of time and in a concrete country by a certain fixed set of diverse indices and indicators. That means, unconditional and objective division of the studied countries by their economic-financial attractiveness using these popular methods is impossible. Here we should note that, in the fundamental work [60], there is a much alike conclusion that contemporary methods of evaluation cannot be applied successfully for unconditional and objective division of countries with notably different level of development by a certain fixed set of diverse indices or indicators. As for us, the main reason behind the absence of relevant methods for unconditional and objective evaluation of economic-financial attractiveness of a country is subjectivity of methods applied for finding weighting coefficients of indices and indicators, which are not given a priori (shall we remind that a weighting coefficient of an index of an indicator reflects its relative importance in the aggregate evaluation; right from that standpoint, weighting coefficient of an index of an indicator is often called "importance coefficient"). Nevertheless, none of thepopular and widely used methods of evaluation of weighting coefficients of indices/indicators/criteria (such as method of weighted sum, method of pointwise estimation, analytic hierarchy process, ranking method, its various modifications, principal component analysis, method of randomized consolidated indices, method of paired comparison sand, its various modifications, Churchman-Ackoff method, Fishburn method, etc.) carries no subjectivity in it.

We are convinced that the above-mentioned subjectivity can be repaired as a problem, if there is a powerful apparatus of the inverse and ill-posed problems theory being used. It has been applied to various applied problems of mathematical physics: as for now, application of that apparatus to the economic and financial analysis, in particular – to the problem of evaluation of economic-financial potential of a country, is just absent.

In order to concretize the above-mentioned subjectivity in the popular approaches for finding weighting coefficients of indices / indicators / criteria, we have to look at the approaches, which have been based on expert judgments, briefly. In the application of the theory of expert evaluations, normally, feasible solution is taken on the ground of correlated experts' opinions (for instance, see [46, 80, 103, 104]), i.e. those experts' opinions in a commission, which do differ notably from the majority's views are being excluded (one should underline that opinions can be non-numerical; for instance, see [81, 102]). That happens, for instance, in some sports in their judgment systems; in the process of taking compromise decisions on economic and financial issues in businesses, where the decision maker (DM) agent is represented by groups (boards of directors, shareholders, etc.). It is evident that, following such approach towards taking acceptable decision (optimal decision cannot even be discussed in such context), when sharply contrasting expert evaluations and opinions are not taken into account, one can easily end up having a blurred final evaluation from a settled probe or expertise, where the extent of being blurred will not be measured in any way, and, moreover, its impact on the final verdict will not be studied. Consequently, such approach does not let minimize impact of blurred expert evaluations on the final DM decision. Moreover, there is one more notable weakness to mention – independently from the kind of approach, which is chosen to evaluate an expertise (is it based on correlated experts' opinions or not), it is emerging on the initial stage of an expertise procedure when members of an expert commission are being selected. Namely, somemembersofthegroupcan:

- unintentionally poorly rate the object of an expertise due to the lack of qualifications; in that case, opinions of such experts, generally, are mutually independent and therefore are not correspondent;
- intentionally poorly rate the object of an expertise targeting other goals, which are not obligatory connected with good execution of that expertise. Suchevaluationsnormallyarecorresponding.

Even if we assume that a selected expert commission consists completely of highly qualified specialists with big experience, who are ideally objective and highly responsible, then the very parameters/indices, whose importance coefficients should be settled by that ideal expert commission, can depend on time for each country in its own way. Moreover, that dependence is not obligatory linear, so, therefore, relevant importance coefficients will also depend on time.

Actually, let the set $\{x_{i,j,k}\}_{i=\overline{1,I};\ j=\overline{1,J}}^{k=\overline{1,K}}$, where through $x_{i,j,k}$ the value of the k-th $(k=\overline{1,K})$ parameter / index is represented for the j-th $(j=\overline{1,J})$ country for the i-th $(i=\overline{1,I})$ year be selection of the unique key parameters / indices; K stands for the number of all unique parameters / indices, which are measured annually; J stands for number of countries whose financial attractiveness is studied; $I=Y_{end}-Y_{start}+1$ stand for number of years when the study is being conducted, starting from the Y_{start} till the year Y_{end} inclusively. Then, the relevant set $W=\{w_{i,j,k}\}_{i=\overline{1,I};\ j=\overline{1,J}}^{k=\overline{1,K}}$ from the importance coefficients / parameters $\{x_{i,j,k}\}_{i=\overline{1,I};\ j=\overline{1,J}}^{k=\overline{1,K}}$ can be introduced as one of the approaches, which are represented below:

(A) $w_{i,j,k}\equiv w=const,$ and then it means that all K parameters / indices are absolutely equally important for all J countries for all the period of time $[Y_{start},Y_{end}]$;

(B) $w_{i,j,k}=w_{j,k},$ and then it means on default that each parameter / index $x_{i,j,k}$ has its own importance, which is constant for all J countries for the period of time $[Y_{start},Y_{end}]$;

(C) $w_{i,j,k}=w_{j,k},$ and then it means on default that each parameter / index $x_{i,j,k}$ can have different importance for each country, but for each of them it will not change for the entire period of time $[Y_{start},Y_{end}]$;

(D) $w_{i,j,k}=w_{i,k},$ and then it means on default that each parameter / index $x_{i,j,k}$ can have different importance in different years, but for all J countries importance of parameters / indices stay the same;

(E) $w_{i,j,k}\in\mathbb{R}^1$, and then it means that for each of the J studied countries the parameter / index $x_{i,j,k}$ has its own importance, which stays unchanged only within the time of a year, but can take different value in a different year;

(F) $w_{i,j,k}=w_{i,j,k}(t),\ t\in Y_i\subset[Y_{start},Y_{end}],$ and then it means that, within the i-*th* $(i=\overline{1,I})$ year, importance of the parameter / index $x_{i,j,k}$ can change: illustrational example from Tab. 1 demonstrates quite possible dynamics of importance coefficient of a certain parameter / index (shall that be the 3rd $x_{i,j,3}$, meaning, for instance GDP (Gross Domestic Product) for the period 2010-2012 in four countries (shall they be Latvia, Estonia, Albania, and Lithuania) whose financial attractiveness is being studied.

We shall look at Tab. 1 briefly. In that table one can see that the parameter GDP, for instance, in January and August 2010, was the most important parameter / index for Latvia, Estonia, and Lithuania, while, in April and May 2011, its importance in these countries had decreased by one position; further, in December 2012, the GDP parameter had the fourth importance for Latvia and Albania among all other parameters / indices, while its importance for Estonia and Lithuania was placed second, etc.

Table 1. An illustrative example of dynamics of the significance coefficient of the GDP parameter in the period of 2010-2012 for the four countries assuming that the parameters are measured monthly: score 1 means the highest significance of the parameter; further down in descending order of significance

<table>
<tr><th rowspan="3">Country</th><th colspan="36">$w_{i,j,3}\ \left(i=\overline{1,3};\ j=\overline{1,4}\right)$</th></tr>
<tr><th colspan="12">2010</th><th colspan="12">2011</th><th colspan="12">2012</th></tr>
<tr><th>Jan.</th><th>Feb.</th><th>Mar.</th><th>Apr.</th><th>May</th><th>Jun.</th><th>Jul.</th><th>Aug.</th><th>Sep.</th><th>Oct.</th><th>Nov.</th><th>Dec.</th><th>Jan.</th><th>Feb.</th><th>Mar.</th><th>Apr.</th><th>May</th><th>Jun.</th><th>Jul.</th><th>Aug.</th><th>Sep.</th><th>Oct.</th><th>Nov.</th><th>Dec.</th><th>Jan.</th><th>Feb.</th><th>Mar.</th><th>Apr.</th><th>May</th><th>Jun.</th><th>Jul.</th><th>Aug.</th><th>Sep.</th><th>Oct.</th><th>Nov.</th><th>Dec.</th></tr>
<tr><td>Latvia</td><td colspan="2">1</td><td>2</td><td colspan="4">3</td><td>1</td><td>3</td><td colspan="6">5</td><td colspan="3">2</td><td colspan="3">4</td><td>1</td><td>5</td><td colspan="2">1</td><td colspan="4">3</td><td>1</td><td colspan="2">3</td><td colspan="4">4</td></tr>
<tr><td>Estonia</td><td colspan="9">1</td><td colspan="22">2</td><td>1</td><td colspan="4">2</td></tr>
<tr><td>Albania</td><td colspan="4">6</td><td colspan="7">5</td><td colspan="3">4</td><td>6</td><td>4</td><td colspan="4">6</td><td colspan="3">5</td><td colspan="11">6</td><td colspan="2">4</td></tr>
<tr><td>Lithuania</td><td>1</td><td>2</td><td>1</td><td>2</td><td>1</td><td>2</td><td colspan="4">1</td><td colspan="7">2</td><td colspan="5">1</td><td colspan="2">2</td><td>1</td><td>2</td><td>1</td><td>2</td><td colspan="7">3</td><td>2</td></tr>
</table>

Now, we are considering the above-mentioned approaches (A)-(F) of introduction of the importance coefficient of a measured parameter / index in rather more detail.

The approach (A) is very widely applied not only in the economic and financial researches. Evidently, while choosing the approach (A) we can assume $w_{i,j,k} \equiv 1.$ Authors of that idea are sure that the approach (A) is eventually leading to blurred results, which risk to become the ground for mistaken decisions and false prediction (both overly optimistic and pessimistic): for example, it is not possible to suggest that the parameters CPI (Corruption Perceptions Index), QLI (Quality of Life Index), FDI (ForeignDirectInvestment) have identical importance both for a highly developed country (from the standpoint of its economic and social indicators) with its nearly inviolable tradition of democratic institutions (for instance, Germany, France, the US, etc.), and for a country, which has acquired independence recently and got on the way of democracy and development of market economy (for example, Serbia, Macedonia, Latvia, Lithuania, Estonia, Poland, Bulgaria, Albania, etc.), and for a country, which suffers civil war and where around 45% of the population are living for less 1 US dollar per day (Somalia). Just as it is not possible to consider that the school subjects of Mathematics, Physics, Dances, or Vocals are equally important for a student (who, for example, has 10, 10, 4, 4 points results in the relevant subjects by the 10-point academic grading system), who is indented to enroll for the Oxford University Faculty of Mathematics programme, and for a student, who(relevant results: 4, 4, 10, 10), who is intended to pursue the career of a street signer after high school: according to the approach (A), all the listed school subjects have equal importance for both students, so they have equal aggregate result averaging 7. Summarizing the foregoing, we can consider the method (A) as the ordinary averaging over the values of all parameters / indices on the aggregate of all arguments.

The approach (B) is also very widely applied, and, again, not only in the economic and financial research. The use of that approach in case of introduction of the importance coefficients of parameters / indices is totally identical with the heuristic method WSM (weighted sum method) in the multi-objective linear programming: the WSM is the most widespread and long-term known as well as the most used up to now method, on whose ground lies the idea of linear convolution of all criteria to one only aggregate criterion represented by the sum of the criteria weighted by the coefficients of their own importance. Despite the attractiveness of the WSM (its attractiveness is mainly motivated by its cognitive simplicity) for many users in execution of calculations (as they do not want to get into the contemporary theory of decision taking due to some or to other reasons), there are notable weaknesses in it with some being unrepairable. Here we speak of the two major of its unrepairable weaknesses [84]. The matter of the first unrepairable weakness of the WSM (and, consequently, of the approach (B)) is in the correspondence of its criteria (parameters / indices in the approach (B)), which by, by its importance, is the same under any variations of the criteria (parameters / indices). In other words, in the WSM, quite real possibility of one criterion's importance's dependence from that of another is ignored fully, while such dependence is pretty often: for example, (see [83, 84]), choosing a summerhouse with some area for summer activities, a renter can assume that having a swimming pool in his summerhouse makes near location of a forest more important than near location of a lake or a river. While, in case if there is no swimming pool located in the territory of the summerhouse, near location of a lake or a river becomes more important than that of a forest. It is evident that in such simple, but quite real example one cannot assume that the importance of each of the two criteria is not changing and does not depends on from the value taken by the other criterion. The matter of the second unrepairable weakness of WSM is in the way how criteria (parameters / indices) $\{x_i\}$ are being given weighting coefficients $\{w_i\}$ without taking into account relevant minimal and maximal values of the criteria in case of many alternatives. That major and unrepairable weakness of WSM is called "intellectual mistake" in the work [25], and that cannot be repaired even applying a sensitivity analysis, i.e. detecting borders of the possible change of criteria, in whose limits the obtained solution stays unchanged. In the work [25], there is a new approach suggested, which is called "SMART" by the author. It, under some additional requirements, executes correct correspondence between importance coefficients and criteria (parameters / indices). Finally, we should also note one more important weakness of the WSM, which is represented in breaking of the Nash's axiom of independence of irrelevant alternatives (see [75] as well [64]); notably, if many alternatives get added (or, in contrast, get excluded) an a priori not the best alternative, then the solution of the problem of finding the best alternative can change. Nevertheless, the mentioned weakness can be avoided, if the arbitration model of Raiffa (see [86] as well [64]) is applied or if one uses the Kalai-Smorodinsky's axiom of monotonicity (see [50] as well [74]), etc. (see [50] as well [16, 20, 59, 91] and respective references given there) instead of the

Nash's axiom and side alternatives. Concluding the above-mentioned, one can consider that the approach (B) has, in addition to its evident advantages, some weaknesses, which can lead towards blurred results and recommendation. That should be taken into account firmly by those who apply the approach (B), as it can lead towards taking quite wrong decisions.

The approaches (C) and (D) are the approaches of one range in the sense that both have the importance coefficients of each parameter / index being dependent from one more natural argument (in addition to the number of parameter / index itself) – either from the number of country (in case of the approach (C)), where the considered parameter / index describes its social political, macroeconomic, financial, etc. condition for the entire period of study, or from year (in case of the approach (D)), when the considered parameter / index was measured in all the countries whoseeconomic-financial attractiveness is being studied. The illustrational example analogical to the one represented in the Tab. 1for the approach (F), would have had a look of a table with horizontal monocoloured stripes (where each country-stripe has its own colour, and, actually, these colours can even be the same sometimes), and, in case of the approach (D), it would have a look of a table with vertical monocolouredstripes (each year-column has its own colour, and sometimes the colours can be same, see Tab. 2): for example, the illustrational example from the Tab. 2 demonstrates one of possible situations with importance coefficient of one parameter / index (as in the Tab. 1, shall it be the GDP parameter again) for the period 2010-2012 in four countries (shall they be again Latvia, Albania, Estonia, and Lithuania), whose economic-financial attractiveness is being studied.

Table 2. An illustrative example describing one of various situations concerning quite possible behaviour of the significance coefficient of the GDP parameter\index in the period of 2010-2012 for the four countries assuming that the parameters\indicesare measured yearly: score 1 means the highest significance of the parameter\index; further down in descending order of significance.

<table>
<tr><th rowspan="3">Country</th><th colspan="36">$w_{i,j,3}\ \left(i=\overline{1,3};\ j=\overline{1,4}\right)$</th></tr>
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<tr><td>Albania</td></tr>
<tr><td>Lithuania</td></tr>
</table>

In the economic-financial studies (generally speaking, also in other applied social economic and social political domains), authors of this work have not encountered, which use the approaches (C) and (D): the only exclusion is represented by the fundamental work [60], which only has some indications and intention on integration of the importance coefficients according to the approaches

(C) and (D). Nevertheless, one should note that the ideas and approaches alike the approach (D) somehow or other in a clear or mutated form are being applied in the domain of online monitoring and diagnostics of dynamic technical (for instance, see [90, 92- 94]), especially in the domain of aerospace engineering (for instance, see [5, 6, 38, 40, 42, 112, 114] and respective references given there) and mechatronics (for instance, see [49, 52, 65, 66] and respective references given there). Concerning the approaches (E) and (F), authors do not know any work, where at least in texted format authors were speaking of introduction of such coefficients for criteria or parameters / indices. Most likely, possibility of introduction of importance coefficients for criteria and parameters / indices by the approaches (E) and (F) is discussed for the first time in this very work. In the context of the considered problem of detection of the economic-financial attractiveness/potential of a group of countries, the approach (F), as for our opinion, does consider all possible situations in relation to the dynamics of changeable parameters/indices and, therefore, it can be considered as the major approach, whose result can be called variety of full weighting coefficients or variety of full importance coefficients. Evidently, the approach (E) is coming out of the approach (F), assuming that measurements of the parameters / indices are conducted once per year (normally, by the end of it): in this case $Y_i = 1$ for $\forall i = \overline{1, I}$, and, therefore, $w_{i,j,k}(t) = w_{i,j,k}(1) = w_{i,j,k}$, i.e. the method (F) goes into the method (E).

So, after having discussed the above-mentioned six approaches of introduction of importance coefficients for parameters / indices, one can ask the following question: which one of them is the best, and which one – the worst? Evidently, the best one is the approach (F), which introduces the set of full importance coefficients. The approach (E), because it is covering different variants of the dynamics of parameters / indices, is quite close to the approach (F), but, in contrast to it, we now have an unstudied idea of finding importance coefficients through the approach (E). As it was already pointed, the approaches (C) and (D) are of one range, and, in this work, the importance coefficients of parameters / indices are introduced through one of them (D). Also evidently that the worst one is the approach (A), whose matter is in finding the arithmetical mean of parameters by all the arguments what these parameters depend from. In relation to the most popular and often used method (B), one can use the words of the quite well-known Soviet Union scientist Dr.Sc.Eng. Professor E.S.Ventsel (08.03.1907 – 15.04.2002): "...arbitrariness carried from one level to the other. ...But manoeuvring with arbitrarily assigned weighting coefficients is called "science"! Intrinsically, there is no science here, and there is nothing to deceive ourselves..."(see [107] as well [63, 84]).

In the current paper, introducing thew eighting coefficients of parameters/ indicesbytheapproach (D), we are studying the problem of evaluation of theeconomic-financial attractiveness of the sixteen countries of the Central and Eastern Europe by 43 of their annually measured social political, macroeconomic, and financial parameters / indices for the period between 2010 and 2016 inclusively. The study has been conducted with the purpose of detection the best-suited countries of these sixteen for exporting banking and other luxurious services there. A it has already been pointed in the begging of this part of the article, the 43 parameters/indices have been selected from the standpoint of the goal settled by us from the big variety of data and factors, traditionally presented in The World Bank's and other comparable transnational organizations' studies. In the work, we have developed a mathematical model, which consists of the importance coefficients whose values are up for detection in addition to its two own unknown "interior" parameters introduced according to the approach (D). Thus, the developed model is an underdetermined parametric model, where one should find all the unknown parameters and the sought-for solution (that solution is the economic-financial attractiveness / potential of each of the sixteen studied Central and Eastern European countries) by the known numerical values of the 43 parameters / indices for the 7 years period 2010-2016. In the work, the apparatus of the theory of inverse and ill-posed problems in finite-dimensional Hilbert spaces is applied for solving the developed parametric model.

2. Statement of the investigated problem and its mathematical model

2.1. Verbal statement of the investigated problem, processing of source statistical data, and creating special types of matrices of parameters on the basis of processed statistical data

Formulating the concept of the studied problem in the generic way, during the period of I years we rate the K key parameters / indices (key social political indicators, macroeconomic performance results, and business environment factors) of the J countries, and, moreover, it is assumed that in the i-th $(i=\overline{1,I})$ year the k-th $(k=\overline{1,K})$ parameter / index of the j-th $(j=\overline{1,J})$ country is rated with one $x_{i,j,k}$, integrally characterizing variety of factors, which indirectly or directly impactthe parameter / index $x_{i,j,k}$. It is required to:

- detect the economic-financial attractiveness / potential of each of the J countries and range it by the aggregate of the found economic-financial potentials;
- detect the "degree of favourability" and the "degree of succession" of each year $i=\overline{1,I}$ both by each of the K parameters / indices and also their aggregate;
- range the years themselves by the mentioned "degree of favourability" and the "degree of succession";
- detect impacts of the years'"degree of succession" on the economic-financial potentials of the studied countries.

The introduced terms of the "degree of favourability" and the "degree of succession" are key important for construction of such mathematical model, which would allow to execute objective evaluation of the economic-financial attractiveness / potential of a country, having the values of the parameters / indices $\{x_{i,j,k}\}$ for I years for all the J countries only and having not any other subjective judgments, suggestions, or probability predictions. The matter of these two new terms is clear from their very names, while the necessity of their introduction is going to become clear from the following workings:

- "degree of favourability" of a year is pointing on the weight / importance of the current year out of the I by their final results;
- under the term "final result" one means the economic-financial potential of a country, which is subjected to evaluation.

Before we switch to the construction of a mathematical model of the above-formulated problem, it is necessary to describe the prestarting procedure, where preparation of the initial statistic data is done $\{x_{i,j,k}\}_{i=\overline{1,I};\ j=\overline{1,J}}^{k=\overline{1,K}}$ for applicability of those in the mathematical model as two-index values and for processing of the initial input data.

In the first step of this prestarting procedure, the values of the parameters / indices taken from the resources listed in [2] should be input to the Tab. 3, whose work region (i.e. table data regions) is the matrix P^{source} of the size $K\times(I\cdot J)$, where the two-index element $p_{a,b}^{\text{source}}\ (a=\overline{1,K};\ b=\overline{1,(I\cdot J)})$ stands for the a-th $(a=\overline{1,K})$ parameter / index for the i-th $(i=\overline{1,I})$ year in the $\left(1+\dfrac{b-i}{I}\right)$-th $(b\equiv i\ (\mathrm{mod}\ I),\ \forall i=\overline{1,I})$ country. Evidently, by the given values of a and b (i.e. on the given cell reference of the Tab. 3) we can uniquely define number of parameter / index, number of year and number of country: for example, if the economic-financial potentials of 16 countries are studied (i.e. J=16) by 43 parameters / indices (i.e. K=43) in the period between 2010 and 2016 inclusively (i.e. I=7), then in the relevant matrix P^{source} of the size $43\times\underbrace{112}_{7\ 16}$ the element $p_{3,17}^{\text{source}}$ represents the value of the third parameter / index (i.e. k=3) for theyear 2012 (i.e. i=3) in the country numbered as 3 (i.e. j=3); the element $p_{42,42}^{\text{source}}$ stands for the 42[nd] parameter / index (i.e. k=42) for 2016 (i.e. i=7) in the country numbered 6 (i.e. j=6); etc.

Table 3. Initial table of source statistical data

Parameters / Indices $\left(k=\overline{1,K}\right)$	Countries $\left(j=\overline{1,J}\right)$									
	Country #1			Country #1			…	Country #J		
	Years $\left(i=\overline{1,I}\right)$			Years $\left(i=\overline{1,I}\right)$				Years $\left(i=\overline{1,I}\right)$		
	Y_{start}	…	Y_{end}	Y_{start}	…	Y_{end}	…	Y_{start}	…	Y_{end}
Parameter #1	value	…	value	value	…	value	…	value	…	value
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Parameter #K	value	…	value	value	…	value	…	value	…	value

Making an important comment, among the values of parameters / indices, there can be zero and negative numbers, and, then,like it happens in the zero-sum matrix games, the payoff array is transformed to the matrix with only positive elements; so, our current matrix P^{source} can be transformed in a same way, using, for example, the linear transformation $\hat{p}^{\text{source}}_{a,b} = p^{\text{source}}_{a,b} + \left|\min_{\substack{n=\overline{1,K} \\ n=\overline{1,(I\cdot J)}}} \{p^{\text{source}}_{n,m}\}\right| + 1$ for $\forall a = \overline{1,K}$ and $\forall b = \overline{1,(I\cdot J)}$. Evidently, from the standpoint of mathematics, the initial and the transformed sets of statistical data are absolutely equivalent and, therefore,all the statistical conclusions in relation to the old $\{p^{\text{source}}_{a,b}\}$ and new $\{\hat{p}^{\text{source}}_{a,b}\}$ values are the same. Therefore, also further, without loss of generality we will keep assuming that $p^{\text{source}}_{a,b} > 0$ for $\forall(a = \overline{1,K};\ b = \overline{1,(I\cdot J)})$.

Continuing the description of the steps of the preparation procedure. In the second step of the prestarting procedure, scaling (in case of need) and standardization of the matrix P^{source} happen. Let's consider the scaling stage. If the matter of a parameter / index is such that its bigger value leads towards lesser attractiveness for exports of banking and other luxurious services to a country (let's call that parameter / index inverse), then it is necessary to conduct homological scaling of that parameter / index in order to get rid of the indicated inverse proportionality. As the result of application of homological scaling, we are getting the new matrix P^{scaled} of the size $K \times (I \cdot J)$,which differs from the matrix P^{source} only by the fact that in some lines (in those which do not correspond with the inverse parameters / indices) of the matrix P^{scaled} all the elements have scaled values.

Now, let's consider the standardization stage. For the realization of it one can use of the approaches, which are disclosed in the fundamental books [1, 22, 37, 88]. Below we are listing just three of them, noting with no further clarifications that the choice what we made by picking up these three of them from many available is not a random.

<u>Standardization approach 1</u>. For standardization of the elements of the matrix P^{scaled} we can apply the following linear transformation: elements a-th $(a = \overline{1,K})$ of the lines of the matrix P^{scaled} are transformed by the formula $p^{\text{scaled\&standardized}}_{a,a+i-1+I\cdot(j-1)} = \dfrac{p^{\text{scaled}}_{a,a+i-1+I\cdot(j-1)}}{\Delta_{a,i}} - \delta_{a,a+i-1+I\cdot(j-1)}$, $\forall(a = \overline{1,K};\ i = \overline{1,I};\ j = \overline{1,J})$, where the value $\Delta_{a,i} = \sup_{m=\overline{1,J}}\{p^{\text{scaled}}_{a,a+i-1+I\cdot(m-1)}\} - \inf_{m=\overline{1,J}}\{p^{\text{scaled}}_{a,a+i-1+I\cdot(m-1)}\}$ stands for a jump of the a-th parameter / index in the i-th year on aggregate of all the J countries, while the value $\delta_{a,a+i-1+I\cdot(j-1)} = \dfrac{\Delta_{a,i} - \varepsilon \cdot p^{\text{scaled}}_{a,a+i-1+I\cdot(j-1)}}{\varepsilon \cdot \Delta_{a,i}}$, $0 < \forall\varepsilon < 1$ stands for the fluctuation of the a-th parameter / index in the i-th in each of these J countries. The use of that approach of standardization generates the new matrix $P^{\text{scaled\& standardized}}$ of the size $K \times (I \cdot J)$, elements $\left\{p^{\text{scaled\&standardized}}_{a,b}\right\}^{b=\overline{1,(I\cdot J)}}_{a=\overline{1,K}}$ definitely are in the segment $[\varepsilon, 1+\varepsilon]$, where $0 < \forall\varepsilon \ll 1$.

Standardization approach 2. For standardization of the elements of the matrix P^{scaled} one can apply the following linear transformation: elements a-th $(a=\overline{1,K})$ of a line of the matrix P^{scaled} are transformed by the formula $p^{\text{scaled\&standardized}}_{a,a+i-1+I\cdot(j-1)} = \dfrac{p^{\text{scaled}}_{a,a+i-1+I\cdot(j-1)} - \inf\limits_{m=\overline{1,J}}\{p^{\text{scaled}}_{a,a+i-1+I\cdot(m-1)}\}}{\Delta_{a,i}}, \ \forall(a=\overline{1,K};\ i=\overline{1,I};\ j=\overline{1,J}),$

where the value $\Delta_{a,i}$ is calculated by the same formula as in the first approach to standardization and carries the same meaning. The use of the current approach generates the new matrix $P^{\text{scaled\&standardized}}$ of the size $K\times(I\cdot J)$, whose elements $\left\{p^{\text{scaled\&standardized}}_{a,b}\right\}_{a=\overline{1,K}}^{b=\overline{1,(I\cdot J)}}$ are in the segment $[0,1]$.

Standardization approach 3. For standardization of the elements of the matrix P^{scaled} one can use a popular approach, which is often called z-values or z-scores: elements of the a-th $(a=\overline{1,K})$ row of the matrix P^{scaled} are transformed in accordance to the formula $p^{\text{scaled\&standardized}}_{a,b} = \dfrac{p^{\text{scaled}}_{a,b} - m_a}{\sigma_a}, \ \forall(a=\overline{1,K};\ b=\overline{1,(I\cdot J)}),$ where $m_a = \dfrac{\sum_{n=1}^{I\cdot J} p^{\text{scaled}}_{a,n}}{I\cdot J}$ and $\sigma_a = \sqrt{\dfrac{\sum_{n=1}^{I\cdot J}(p^{\text{scaled}}_{a,n} - m_a)^2}{I\cdot J}}$ stand for the mean value and the standard deviation of the a-th parameter / index all over the J countries and I years, respectively. As the result of application of that approach, we are getting the new matrix $P^{\text{scaled\&standardized}}$ of the size $K\times(I\cdot J)$.

In conclusion of the second step of the preparation procedure, we would like to note the following: in the second standardization approach, unlike the case of the first one, the space-time fluctuations of the parameters / indices are not taken into account. From that standpoint, application of the first standardization approach, as for us, appears to be better idea than that of the second one. In relation to the third standardization approach, we should only add that due to many reasons it is notably less suited for application to the studied problem than the two others.

The third and the last step of the preparation procedure is construction of the matrix of parameter for each of the K parameters / indices: notably, the matrix of the k-th $(k=\overline{1,K})$ parameter / index is our matrix $P^{\text{basic}}(k)$ of the size $I\times J$, where the element $p^{\text{basic}}(k)_{i,j} = p^{\text{basic}}_{i,j,k}$ standing for the value of the k-th $(k=\overline{1,K})$ parameter / index for the i-th $(i=\overline{1,I})$ year in the j-th $(j=\overline{1,J})$ country is calculated by the formula $p^{\text{basic}}_{i,j,k} = p^{\text{scaled\&standardized}}_{k,i+I\cdot(j-1)}$.

Thus, as the result of execution of the standardization procedure, we are getting exactly K of the matrixes $P^{\text{basic}}(k)$, where each of them has the size of $I\times J$. Further, we will use only the matrixes $P^{\text{basic}}(k),\ k=\overline{1,K}$, and when we speak of the input data, we assume these K matrixes of parameters.

2.2. Mathematical model of the stated problem

In the current subsection, there is a mathematical model formulated in the subsection 2.1 of the problem being suggested. Detailed interpretation of all the variables and parameters, which are mentioned in the suggested mathematical model, lets understand the course of construction of this mathematical model as well as comprehend the goals of introduction of the variables and parameters, which do have place in the model.

The suggested model gets the following form:

$$X(k) = \lambda \cdot (P^{basic}(k))^T Y(k),\ \lambda \in \mathbb{R}^1_{++},\ X(k) \in \mathbb{R}^{J\times 1}_{++},\ Y(k) \in \mathbb{R}^{I\times 1}_{++},\ \forall k = \overline{1,K}, \tag{1}$$

$$Y(k) = \|Y(k)\|_\infty - \mu R(k) U(J),\ \mu \in (0,1],\ U(J) \in \mathbb{R}^{J\times 1}_{++},\ R(k) \in \mathbb{R}^{I\times J},\ \forall k = \overline{1,K}, \tag{2}$$

$$(P^{\mathrm{basic}}(k))^T W(k) = X(k),\ W(k) \in \mathbb{R}^{I\times 1}_{++},\ \forall k = \overline{1,K}, \tag{3}$$

$$P^{average} Z = \frac{I}{J} \cdot X^{average},\ P^{average} \in \mathbb{R}^{J\times K}_{++},\ Z \in \mathbb{R}^{K\times 1}_{++},\ X^{average} \in \mathbb{R}^{J\times 1}_{++}, \tag{4}$$

where

- $\mathbb{R}^{n\times 1}_{++} = \{a = (a_1, a_2, \ldots, a_n)^T : a_i > 0\ \forall i = \overline{1,n}\}$; designation $A \in \mathbb{R}^{m\times n}$ means that A is matrix of the size $m \times n$, whose elements can be any real numbers; designation $A \in \mathbb{R}^{m\times n}_{++}$ means that A is matrix of the size $m \times n$, whose elements can be positive numbers only.
- $P^{\mathrm{basic}}(k) \in \mathbb{R}^{I\times J}_{++},\ k = \overline{1,K}$ is matrix of the k-th parameter / index, and for each of the K parameters / indices it is the result of implementation of the prestarting procedure, which consists of the three steps; they are fully disclosed in the subsection 2.1.
- $X(k) \in \mathbb{R}^{J\times 1}_{++},\ k = \overline{1,K}$ is column-vector, an element $x(k)_j$ $(j = \overline{1,J})$ of which stands for the sought-for economic-financial attractiveness / potential of the j-th $(j = \overline{1,J})$ country by the k-th parameter / index.
- $Y(k) \in \mathbb{R}^{I\times 1}_{++},\ k = \overline{1,K}$ is vector-column, whose element $y(k)_i$ stand for the sought-for "degree of favorability" of the i-th $(i = \overline{1,I})$ year by the k-th parameter / index.
- $\|Y(k)\|_\infty = \sup\limits_{i=\overline{1,I}} |y(k)_i| = \max\limits_{i=\overline{1,I}} y(k)_i$.
- $U(\ ) \in \mathbb{R}^{n\times 1}_{++}$ is a constant vector-column, all elements of which are equal to 1. In particular, we will use $U(I), U(J)$, etc.
- $R(k) \in \mathbb{R}^{I\times J},\ k = \overline{1,K}$ is a matrix of residuals, whose element $r(k)_{i,j}$ $(i = \overline{1,I};\ j = \overline{1,J})$ is defined as $r(k)_{i,j} = \left| \underbrace{p^{\mathrm{basic}}(k)_{i,j}}_{p^{\mathrm{basic}}_{i,j,k}} - x(k)_j \right|$, $(i = \overline{1,I};\ j = \overline{1,J})$ and stands for the sought-for "degree of succession" of the i-th $(i = \overline{1,I})$ year in relation to the j-th $(j = \overline{1,J})$ country by the k-th parameter / index. Then,

the i-th $(i=\overline{1,I})$ element of the vector-column $R(k)U(J)$, which is equal to $\sum_{j=1}^{J}\left|p^{\text{basic}}(k)_{i,j}-x(k)_j\right|$, can be interpreted as sum evaluation, which is given by the i-th year to all the J countries depending on how the k-th parameter / index influenced the economic-financial attractiveness of each of the J studied countries. Therefore, the vector-column $R(k)U(J)$ can be interpreted doubly: as aggregate of the "degrees of succession" of years by the k-th parameter / index, or as aggregate of the "degrees of consistency" of years by the k-th parameter / index. Consequently, first of all, everywhere instead of "degree of succession" we can use "degree of consistency"; secondly, by the values of elements of all K vector-columns $R(k)U(J)$ we can detect "troublesome years" as well as "troublesome parameters / indices" for each of the J studied countries, which are the reasons why a given country has low economic-financial attractiveness / potential from the standpoint of the possibility to establish exports of banking and other luxurious services there.

– $W(k)\in\mathbb{R}_{++}^{I\times 1},\ k=\overline{1,K}$ is vector-columns, whose element $w(k)_i\ (i=\overline{1,I})$ stands for the sought-for significance coefficient of the k-th parameter / index in the i-th $(i=\overline{1,I})$ year. As it was already mentioned in the section 1 (see the approach (D)), in the current work, we suggest that each parameter / index $p^{\text{basic}}(k)_{i,j}=p^{\text{basic}}_{i,j,k}\in P^{\text{basic}}(k)$ has its own weighting coefficient $w(k)_i=w_{i,j,k}\in W(k)$, so we assume that each parameter / index can have different importance in different years, but for all the J countries they remain unchanged.

– $P^{\text{average}}\in\mathbb{R}_{++}^{J\times K}$ is a matrix, whose element $p^{\text{average}}_{j,k}\ (j=\overline{1,J};\ k=\overline{1,K})$ is defined as $p^{\text{average}}_{j,k}=(U(I))^T(P^{\text{basic}}(k))^{\langle j\rangle},\ (j=\overline{1,J};\ k=\overline{1,K})$, where $(P^{\text{basic}}(k))^{\langle j\rangle}$ is the j-th column of the matrix $P^{\text{basic}}(k)$. In other words, $\frac{1}{I}\cdot p^{\text{average}}_{j,k}$ stands for the averaged value of the k-th parameter / index of the j-th country by the aggregate of values what that parameter / index has over the period of I years.

– $X^{\text{average}}\in\mathbb{R}_{++}^{J\times 1}$ is vector-column, whose element $x^{\text{average}}_j\ (j=\overline{1,J})$ is defined as $x^{\text{average}}_j=\sum_{k=1}^{K}x(k)_j$, i.e. an element $\frac{1}{J}\cdot x^{\text{average}}_j$ is an averaged value of the economic-financial attractiveness of the j-th country by the aggregate of all parameters / indices.

– $Z\in\mathbb{R}_{++}^{K\times 1}$ is a vector-column, whose element $z_k\ (k=\overline{1,K})$ stands for the "coefficient of relative strengthening" of the importance of the k-th $(k=\overline{1,K})$ parameter / index from the standpoint of two factors: based on the space-time aggregate (that is to say, by all the J countries over all the I years) of the given values of the k-th $(k=\overline{1,K})$ parameter / index; based on the unknown values of the economic-financial attractiveness / potential of each of the J countries over I years. The concept developed by us, "coefficientof relative strengthening" of importance of a parameter / index perhaps is not a successful concept for application towards the problem studied in the current work: we have introduced that concept as an analogy for the concept of coefficient strengthening, which is used in the papers [92], [93]. Therefore, it is necessary to clarify the meaning and goal of introduction of that concept. Assuming that we have found $\{W(k)\}_{i=\overline{1,K}}=\{w(k)_i\}_{i=\overline{1,I}}^{i=\overline{1,K}}$, so the importance coefficients of all the K parameters / indices in each of the I years are found, and these coefficients are the same for all the J countries whose economic-financial attractiveness / potential is studied. Nevertheless, in reality, we cannot claim that: some parameter / index can appear as the most important in a selected year for one country, while for another the same parameter / index can have very little role in the very same year. For instance, we cannot claim that for two same class long-range strike fighter jets, whose full functionality depends, for example, from the qualitative condition of 10000 key technical mechanisms and details, a selected mechanism (say, autopilot system) has the same degree of importance in a selected period of time, independently from the regime and circumstance where both jets of the same class happen to operate at that given period of time. Then, how can we claim that the same economic or political parameter / index for two different countries carries equal importance in a same year? Indeed, we can come up with many other examples in the domains of technics, sociology, finance, economics, ecology, etc. to illustrate the very same example. We cannot claim that in cases of monitoring and / or diagnostics of technical, social, financial, economic, ecological, or any other processes, phenomena, objects, or systems. While introducing importance of indicator through one of the (A)-(E) approaches, which are described in the introduction of the current paper, we are accepting the above-mentioned mistaken assumption (corresponding with the introduction of importance coefficient according to the approach (D) or some other not fully correct assumptions on default. Since, in this paper, introduction of importance coefficient is done according to the approach (D), not according to the most adequate approach (F), whose application's result gives variety of full weighting coefficients, in order to differ individual strength of influence of each parameter / index for each country, considering the pattern of its aggregate economic-financial attractiveness / potential (the study is done with the purpose of establishing exports of banking or other luxurious services), we are introducing the vector-column $Y(k)\in\mathbb{R}_{++}^{I\times 1},\ k=\overline{1,K}$ as well as the matrix $R(k)\in\mathbb{R}^{I\times J},\ k=\overline{1,K},$ whose matter is already clarified above; also we introduce the vector-column

$Z \in \mathbb{R}_{++}^{K\times 1}$. The purpose of the Z is in the partial compensation for absence of full information about the importance coefficientsof parameters / indices, that is to say, for absence of the full set of data $\{w_{i,j,k}(t)\}_{i=\overline{1,I},\, j=\overline{1,J}}^{k=\overline{1,K}}$, $\forall t \in [Y_{start}, Y_{end}]$. Element z_k $(k=\overline{1,K})$ of the introduced vector-column Z, what we labeled as "coefficient of relative strengthening" of importance of the k-th $(k=\overline{1,K})$ parameter / index, can be interpreted as the weight of averaged weight (weight averaging happens only by years, as weights have same values by countries) of that k-th $(k=\overline{1,K})$ parameter / index under the condition of presence of information about the values of $x(k)_j$, $\forall j=\overline{1,J}$, which means that there are values of the economic-financial attractiveness of all the J countries by the k-th $(k=\overline{1,K})$ parameter / index. Consequently, vector-column $Z \in \mathbb{R}_{++}^{K\times 1}$ consists of weights of the importance weights of all the parameters / indices, and that vector-column could not have been constructed without knowledge of the economic-financial attractiveness of the countries by all K parameters / indices.

- Controlled parameter $\lambda \in \mathbb{R}_{++}^{1}$ stands for the coefficient of proportionality of the sought-for economic-financial attractiveness / potential of each of the J studies countries towards the weighted sum of parameters / indices of all the studied countries, i.e. towards the sum $\sum_{i=1}^{I} y(k)_i \cdot p^{\text{basic}}(k)_{i,j} = (P^{\text{basic}}(k))^T Y(k)$. Since the purpose of introduction of the controlled parameter λ is in normalization of the evaluations $x(k)_j$, $j=\overline{1,J}$, it can be chosen selectively: for example, in the current paper, $\lambda = J = 16$, which stands for the number of the studied countries.
- Controlled parameter $\mu \in (0,1]$ stands for the coefficient of sensitivity of the sought-for "degree of succession" of a year (in other words, "degree of consistency" of a year) towards the sought-for "degree of favourability" of the same year. Under the condition of increase of the value of the parameter μ, correlation between the economic-financial attractiveness / potential of a country and the values of parameters / indices of that country over period of I years grows. Value of the parameter μ can be chosen selectively from the range of $(0,1]$, obligatory respecting the $I \cdot K$ conditions $y(k)_i > 0$ for $\forall i=\overline{1,I}$ and $\forall k=\overline{1,K}$.

Thus, in mathematical model (1)-(4), there are $I \cdot J \cdot K$ known data $\{p^{\text{basic}}(k)_{i,j}\}_{i=\overline{1,I},\, j=\overline{1,J}}^{k=\overline{1,K}}$, so the goal is in finding $(2\cdot I + J + 1)\cdot K$ of the unknown data $\{x(k)_j\}_{j=\overline{1,J}}^{k=\overline{1,K}}$, $\{y(k)_i\}_{i=\overline{1,I}}^{k=\overline{1,K}}$, $\{w(k)_i\}_{i=\overline{1,I}}^{k=\overline{1,K}}$, $\{z_k\}_{k=\overline{1,K}}$ by that known input data (we have excluded controlled parameters λ and μ from the unknown, as their values are selected without actual use of the input data).

Then, we briefly address the peculiarities of the structure of the suggested model (1)-(4), starting the consideration from the finite-dimensional operator equation of the first kind (4).

On a peculiarity of (4). Equation (4) represents a system of linear algebraic equations with J equations and K sought-for variables, which are located in the left-hand side of the system. Since, in the current paper, $J < K,$ our system (4) is an underdetermined system. If we introduce the notations $A = P^{\text{average}},\ q = Z,$ $f = \frac{I}{J} \cdot X^{\text{average}},$ then the equation (4) can be rewritten in the following common form:

$$Aq = f, \tag{5}$$

where, as a rule, the matrix A and the vector f are considered to be given input data, while q is considered to be sought-for vector.

Nevertheless, in the operator equation of the first kind (5) (that is to say, in the equation (4), which is written in the form (5)) the right-hand side f also is an unknown, since $f = \frac{I}{J} \cdot X^{\text{average}} = \frac{I}{J} \cdot \left\{ \sum_{k=1}^{K} x(k)_j \right\}_{j=\overline{1,J}}$. Consequently, the right-hand side f of the operator equation (5) will be known only after solving the equation (1), (2).

On a peculiarity of (3). Equation (3) represents K systems of linear algebraic equations, each of that contains J equations and I sought-for variables, which are located in the left-hand side. Since, in the current paper, $J > I,$ each system in (3) appears to be an overdetermined system. If we introduce the notations $A(k) = (P^{\text{basic}}(k))^T,\ q(k) = W(k),\ f = X(k),$ then each of these K systems can also be rewritten in the form of an operator equation of the first kind (5):

$$A(k)q(k) = f(k),\ \forall k = \overline{1,K}. \tag{6}$$

As in the case (4), the right-hand side $f(k)$ of each equation in (6) will be known only after solving the equation (1), (2).

On a peculiarity of (1), (2). Under each fixed k $(k = \overline{1,K})$ in the both operator equations (1) and (2), simultaneously there are different sets of the unknowns: the unknowns $\{y(k)_i\}_{i=\overline{1,I}} = Y(k),$ which are located in the left-hand sideof (2), are appearing in the right-hand sideof (1); the unknowns $\{x(k)_j\}_{j=\overline{1,J}} = X(k),$which are located in the left-hand sideof (1), are appearing (they are in the matrix $R(k)$) in the right-hand sideof (2). In other words, for each fixed k $(k = \overline{1,K})$, the equations (1), (2) form the system of $(I + J)$ interrelated equations. Therefore, they cannot be represented in the form of an operator equation of the first kind (6) (or (5)), for finding the stable solution of what, in the section 3 of the current paper, there is the inverse and ill-poised problems apparatus in finite-dimensional Hilbert spaces being applied.

Thus, in operator equations (3) and (4) the input data, in addition to the already known data $\{p^{\text{basic}}(k)_{i,j}\}_{i=\overline{1,I},\, j=\overline{1,J}}^{k=\overline{1,K}}$, are the outcoming data of the systems (1), (2). Consequently, one can perform study of the operator equations (3) and (4) only after having studied the K systems, each of what consists of $(I+J)$ interrelated equations. In the section 3, there is an iterative process being developed; it lets find stable solution for each of the K systems' $(I+J)$ interrelated equations (1), (2).

2.3.The existence and uniqueness of the solution of the formulated mathematical model (1)-(4)

As it has been mentioned in the subsection 2.2, under each fixed k $\left(k=\overline{1,K}\right)$ the equations (1), (2) are forming system of the $(I+J)$ interrelated equations. In the current subsection, issues of solvability as well as uniqueness of the solution of the system (1), (2) are studied under each fixed k $(k=\overline{1,K})$.

The study is started from the notion of uniqueness of the solution: we prove that system (1), (2) cannot have more than one solution. Assuming the contrary, that is to say, we assume that the pair $\{\overline{X}(k),\overline{Y}(k)\}$ and $\{\overline{\overline{X}}(k),\overline{\overline{Y}}(k)\}$ are two different solutions of the system (1), (2):

$$\begin{cases} \overline{X}(k)=\lambda\cdot(P^{\text{basic}}(k))^T\,\overline{Y}(k), \\ \overline{Y}(k)=\left\|\overline{Y}(k)\right\|_\infty-\mu\cdot\overline{R}(k)U(J), \end{cases} \tag{7}$$

$$\begin{cases} \overline{\overline{X}}(k)=\lambda\cdot(P^{\text{basic}}(k))^T\,\overline{\overline{Y}}(k), \\ \overline{\overline{Y}}(k)=\left\|\overline{\overline{Y}}(k)\right\|_\infty-\mu\cdot\overline{\overline{R}}(k)U(J), \end{cases} \tag{8}$$

where $\overline{X}(k)\not\equiv\overline{\overline{X}}(k)$ and $\overline{Y}(k)\not\equiv\overline{\overline{Y}}(k)$ under each fixed k $(k=\overline{1,K})$.

Having rewritten the systems (7) and (8) in the form

$$\overline{X}(k)=\lambda\cdot(P^{\text{basic}}(k))^T\,\overline{Y}(k)\left(\left\|\overline{Y}(k)\right\|_\infty-\mu\cdot\overline{R}(k)U(J)\right),$$

$$\overline{\overline{X}}(k)=\lambda\cdot\left(P^{\text{basic}}(k)\right)^T\,\overline{\overline{Y}}(k)\left(\left\|\overline{\overline{Y}}(k)\right\|_\infty-\mu\cdot\overline{\overline{R}}(k)U(J)\right),$$

and then, having carried out elementary transformations, we get

$$D(k)\overline{X}(k)=\overline{B}(k), \tag{9}$$

$$D(k)\overline{\overline{X}}(k)=\overline{\overline{B}}(k), \tag{10}$$

where

- $D(k)$ is a quadratic matrix of the size of $J\times J$, whose elements are calculated in accordance to the formula

$$D(k)_{i,j}=\begin{cases} \mu\cdot\left(\left(P^{\text{basic}}(k)\right)^{\langle i\rangle}\right)^T U(I)-\dfrac{1}{\lambda} & \text{if} \quad j=i, \\ \mu\cdot\left(\left(P^{\text{basic}}(k)\right)^{\langle i\rangle}\right)^T U(I) & \text{otherwise}, \end{cases}$$

so, in each row of the matrix $D(k)$, all the elements besides the diagonal element do coincide; consequently, the matrix $D(k)$ is a nondegenerate matrix);

– $\bar{B}(k)$ and $\bar{\bar{B}}(k)$ are vector-columns of the size of $J\times 1$, the elements of which are calculated by the following formulas, respectively:

$$\begin{cases} \bar{b}(k)_j = \left\|\bar{Y}(k)\right\|_\infty \cdot \left(\left(P^{\text{basic}}(k)\right)^{\langle j\rangle}\right)^T U(I) - \mu\cdot\left(\left(P^{\text{basic}}(k)\right)^{\langle j\rangle}\right)^T Q(k),\ \forall j=\overline{1,J}, \\ \bar{\bar{b}}(k)_j = \left\|\bar{\bar{Y}}(k)\right\|_\infty \cdot \left(\left(P^{\text{basic}}(k)\right)^{\langle j\rangle}\right)^T U(I) - \mu\cdot\left(\left(P^{\text{basic}}(k)\right)^{\langle j\rangle}\right)^T Q(k),\ \forall j=\overline{1,J}, \end{cases} \tag{11}$$

where $Q(k)$ is a vector-column of the size of $I\times 1$, the elements of what are calculated by the formula $q(k)_i = \left(\left(\left(P^{\text{basic}}(k)\right)^T\right)^{\langle i\rangle}\right)^T U(J),\ \forall i=\overline{1,I}.$

In the left-hand side of operator equations (9) and (10), there is the same matrix $D(k)$, while in the right one, as that can be seen from (11), the difference consists only of $\left\|\bar{Y}(k)\right\|_\infty$ and $\left\|\bar{\bar{Y}}(k)\right\|_\infty$. Therefore, difference of the operator equations (9) and (10) does let get the sufficient condition, whose application secures $\bar{X}(k)=\bar{\bar{X}}(k)$, and, therefore, $\bar{Y}(k)=\bar{\bar{Y}}(k)$. Actually, taking from the equation (9) the equation (10), we get

$$D(k)\left(\bar{X}(k)-\bar{\bar{X}}(k)\right) = \left(\left\|\bar{Y}(k)\right\|_\infty - \left\|\bar{\bar{Y}}(k)\right\|_\infty\right)\cdot\left(\left(P^{\text{basic}}(k)\right)^{\langle j\rangle}\right)^T U(I).$$

Since the quadratic matrix $D(k)$ is a nondegenerate matrix, then under $\left\|\bar{Y}(k)\right\|_\infty = \left\|\bar{\bar{Y}}(k)\right\|_\infty$ we get homogenous finite-dimensional operator equation $D(k)\left(\bar{X}(k)-\bar{\bar{X}}(k)\right)=0$, which has only the trivial solution $\bar{X}(k)-\bar{\bar{X}}(k)\equiv 0$, i.e. $\bar{X}(k)=\bar{\bar{X}}(k)$. Considering that fact in systems (7) and (8), we get $\bar{R}(k=\bar{\bar{R}}(k)$, and, consequently, we have $\bar{Y}(k)=\bar{\bar{Y}}(k)$. The proof of the uniqueness of the solution $\{X(k);Y(k)\}=\{x(k)_1,\ldots,x(k)_J;y(k)_1,\ldots,y(k)_I\}$ of system (1), (2) under each fixed k $(k=\overline{1,K})$ is fully completed. Nevertheless, there is sense to add on the issue of the condition $\left\|\bar{Y}(k)\right\|_\infty = \left\|\bar{\bar{Y}}(k)\right\|_\infty$, the execution of what does secure uniqueness of the solution of system (1), (2).

Firstly, by virtue of the Fredholm Alternative (for instance, see [51]), this sufficient condition also appears to be a necessary condition for the uniqueness of the solution of system (1), (2). From the very same Fredholm Alternative, emerges the solvability of the system (1), (2) under the condition $\left\|\bar{Y}(k)\right\|_\infty = \left\|\bar{\bar{Y}}(k)\right\|_\infty$.

Secondly, from the interpretation of the value $Y(k)$ (see the subsection 2.2), it is clear that the condition $\left\|\bar{Y}(k)\right\|_\infty = \left\|\bar{\bar{Y}}(k)\right\|_\infty$ means that for the k-th parameter / index the maximum of the "degree of favourability" of all the I years should be an absolute constant (in the sense that this maximum should not depend from the values $\{y(k)_i\}_{i=\overline{1,I}}$, i.e. from the values of the "degree of favourability" of years by the k-th parameter / index). At first glance, that looks paradoxical, but, as it is explained in the section 3, as $\|Y(k)\|_\infty$ one can take quite small absolute constant (for instance, in this paper we have settled on $\|Y(k)\|_\infty = I \cdot J \cdot K = 4816$).

3. Development of method for solving the formulated problem (1)-(4)

3.1. Development of an iterative process for solving the system (1), (2)

So, under each fixed k $(k=\overline{1,K})$ the system (1), (2), which represents the system from the $(I+J)$ interrelated equations, is solvable and has the unique solution $\{x(k)_1,\ldots,x(k)_J; y(k)_1,\ldots,y(k)_I\}$. Below there is an iterative process suggested in order to find solution of the compatible (1), (2) under each fixed k $(k=\overline{1,K})$.

$$\begin{cases} X^0(k)=U(J);\ Y^0(k)=Y_{\max}(k); \\ X^n(k)=\lambda\cdot\dfrac{\left(P^{\text{basic}}(k)\right)^T Y^{n-1}(k)}{\left\|\left(P^{\text{basic}}(k)\right)^T Y^{n-1}(k)\right\|_\infty},\ \forall n\in\mathrm{N}; \\ Y^n(k)=\|Y(k)\|_\infty-\mu\cdot R^n(k)U(J),\ \forall n\in\mathrm{N}, \end{cases} \tag{12}$$

where $Y_{\max}(k)\in\mathbb{R}^I_{++}$ is constant vector-column, all elements of what equal with $\|Y(k)\|_\infty$; $X^0(k), Y^0(k)$ mean initial approximation, while $X^n(k), Y^n(k)$ stand for the n-th approximation.

Before the issue of convergence and iterative process is addressed, we make the following important note: the initial approximation in the iterative process (12) requires knowledge of the value $\|Y(k)\|_\infty=\max\limits_{i=\overline{1,I}} y(k)_i$ for $\forall k=\overline{1,K}$. Since $\{y(k)_i\}_{i=\overline{1,I}}$ are the unknowns for $\forall k=\overline{1,K}$, then $\|Y(k)\|_\infty, \forall k=\overline{1,K}$, and, consequently, $Y_{\max}(k), \forall k=\overline{1,K}$ cannot be known a priori. Nevertheless, due to the fact that we have proved the unconditional convergence of the iterative process (12) in relation to selection of both initial approximation and controlled parameters $\lambda\in\mathbb{R}^1_{++}, \mu\in(0,1]$ (in the current paper, we have opted for $\lambda=J,\ \mu=0.5$), then, logically, it is clear that we can take quite big number as $\|Y(k)\|_\infty$, for example, in this paper, we have opted for $\|Y(k)\|_\infty=I\cdot J\cdot K=4816.$ Since we have not studied rate of convergence of the iterative process (12) , the issue of the influence of selection of initial approximation on the rate of convergence remains open in this paper. In our current work, the issue of stability of the solution of the system (1), (2) (stability against the relatively small changes in the input data) by the iterative method (12) is not studied either.

Now, we are briefly addressing the issue of unconditional convergence of the suggested iterative process (12). The proof of the convergence (12) in relation towards choice of the controlled parameter μ, for example, can be executed by perturbative approach: first, one study the trivial case $\mu=0$, and, then, it can be elementary proved that there is convergence of the iterative (12) in the norm $\|\cdot\|_{L_2}$; then, the trivial case is initially perturbed (i.e. $\mu=\delta,\ 0<\delta\ll 1$, and gets proved that all the discrete functions, which take part in (12) depend on the parameter μ (important role in securing that stable dependence is played by the positivity of the matrixes $P^{\text{basic}}(k),\ k=\overline{1,K}$; see the subsection 2.1), and, therefore, the iterative process (12) coincides under $\delta\to 0$. Applying the analogical approach, one can prove unique convergence (12) in relation towards selection of the initial approximation as well as the λ.

3.2. Development of aregularization algorithm for solving the operator equation of the first kind

As it was reflected in subsection 2.2, mathematical model (1)-(4) decomposes in the following elements:

- K systems, each of what consists of the interrelated equations (1), (2), for the solution of what one can apply the iterative process (7) and find $\{X(k);Y(k)\}=\{x(k)_1,\ldots,x(k)_J;y(k)_1,\ldots,y(k)_I\}$;
- finite-dimensional operator equation of the first kind (5), which appears to be anunderdetermined system of linear algebraic equations, part of the input data of what does consist from the input K of the systems (1), (2);
- K operator equations of the first kind (6), each of what under the fixed k $(k=\overline{1,K})$ gets the form (5), but, unlike (5), is an overdetermined system of linear algebraic equations, part of the input data of what, as in (5), is the input data from the K systems (1), (2).

In the current subsection, the Tikhonov regularization method is addressed, as it lets find stable solution of a determined or undeterminedfinite-dimensional operator equation of the first kind and, therefore, of the equation(5) (which corresponds with the equation (4) of the suggested mathematical model) and the equation (6) (which corresponds with the equation (3) of the suggested mathematical model).

Due to the fact that, in the equations (5) and (6), the relevant principal operators and right-hand sides have different sizes, in the current section, we consider the general equation

$$Aq=f,\ A\in\mathbb{R}_{++}^{m\times n},\ q\in Q\subseteq\mathbb{R}_{++}^{n\times 1},\ f\in F\subseteq\mathbb{R}_{++}^{m\times 1},\ A:Q\to F. \qquad (13)$$

It is evident that:

- when an underdetermined system (5) is studied, then, in general equation (13), there are the following notations:

$$n=J,\ m=K,\ A=P^{\text{average}}=\left\{(U(I))^T\left(P^{\text{basic}}(k)\right)^{\langle j\rangle}\right\}_{j=\overline{1,n}}^{k=\overline{1,m}},\ q=Z=\{z_k\}_{k=\overline{1,m}},$$

$$f=\frac{I}{J}\cdot X^{\text{average}}=\frac{I}{J}\cdot\left\{\sum_{k=1}^{m}x(k)_j\right\}_{j=\overline{1,n}};$$

– when we study an overdetermined system under each fixed k $(k=\overline{1,K})$, then, in the general equation (13), there are the following notations: $n=J,\, m=I,\, A=A(k)=\left(P^{\text{basic}}(k)\right)^T=\left\{p^{\text{basic}}(k)_{i,j}\right\}_{i=\overline{1,n}}^{j=\overline{1,m}},\, q=W(k)=\left\{w(k)_i\right\}_{i=\overline{1,m}},$ $f=X(k)=\left\{x(k)_j\right\}_{j=\overline{1,n}}.$

So, till the very end of the section 3, we are working with the general equation (13), assuming that $A\in\mathbb{R}_{++}^{m\times n}$ and $f\in F\subseteq\mathbb{R}_{++}^{m\times 1}$ are the given input data, $q\in Q\subseteq\mathbb{R}_{++}^{n\times 1}$ is the sought-for solution.

3.2.1. The classical Tikhonov regularization method and choosing of ambiguity of regularization parameter

Let the principal operator A and the right-hand side f of the equation (13) are given exactly. Then, replacing A and f in (1) by A^{exact} and $f^{\text{ exact}}$, respectively we can write

$$A^{\text{exact}}q^{\text{exact}} = f^{\text{ exact}}. \tag{14}$$

Note that the equation (14) can be uniquely solvable (i.e. have a unique solution) or degenerate (either have infinitely many solutions, or be unsolvable): the matrix equation (14) is said to be degenerate, if the determinant of the system is equal to zero, i.e. $\det(A^{\text{exact}})=0,$ however, the matrix A^{exact} may be nondegenerate, but ill-conditioned, i.e., if the condition number of the matrix is large enough: $\mu(A^{\text{exact}})=\dfrac{\lambda_{\max}}{\lambda_{\min}}>1,$ where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues of the matrix $A^{\text{exact}},$ respectively. It should be noted that, if the calculations are executed with finite precision, in some cases, it is not possible to determine whether a given system of equations is singular or ill-conditioned. In other words, singular and ill-conditioned systems can be indistinguishable within the specified accuracy.

A pseudosolution of the system (14) is a vector $\tilde{q}$ that minimizes the residual $\|Aq-f\|$ on the whole space $\mathbb{R}^n,$ and this pseudosolution may be not unique. Then, we denote by Q_A a set of all the pseudosolutions of the system (14). In this case, the normal solution of the system (14) is called a pseudosolution q^0 with minimal norm $\left\|q^{\text{normal}}\right\|_{\ell_2},$ i.e. such as $\left\|q^{\text{normal}}\right\|_{\ell_2}=\inf\limits_{q\in Q_A}\|q\|_{\ell_2}.$

Due to the inaccuracy of the measurement equipment, it is impossible to determine the absolutely precise true values of the parameters A_{exact} and $f^{\text{ exact}},$ we get distorted values, which we denote by $A^h\in\mathbb{R}_{++}^{m\times n}$ and $f^{\delta}\in\mathbb{R}_{++}^{\delta}$ respectively. Nevertheless, the experimenter usually does determine his maximum range of deviation, which gives us information about how the values A^h and f^{δ} are close to the true ones. In other words, we know some values $h\in\mathbb{R}_{+}^{1}$ and $\delta\in\mathbb{R}_{++}^{1},$ which satisfy the inequalities:

$$\left\| A^h - A^{\text{exact}} \right\|_{\ell_2(Q)} \le h, \ \left\| f^{\delta} - f^{\text{ exact}} \right\|_{\ell_2(F)} \le \delta. \tag{15}$$

Hence, instead of (14), in fact, we have a different equation

$$A^h q^{\{h,\delta\}} = f^{\delta}, \tag{16}$$

whose solution is another vector $q^{\{h,\delta\}} \in \mathbb{R}^{n\times 1}_{++}$, which may considerably differ from the exact solution q^{exact} of the equation (14). Therefore, we need to find a solution $q^{\{h,\delta\}} \in Q$ of the equation (16) on the set $\left\{A^h, f^{\delta}; h, \delta\right\}$, satisfying the inequality conditions (15), and this solution has to be stable, i.e. $\left\| q^{\text{normal}} - q^{\{h,\delta\}} \right\|_{\ell_2(Q)} \xrightarrow{\delta\to 0,\ h\to 0} 0$, where by $q^{\text{normal}} \in Q$ we denote normal pseudosolution (solution with minimal norm in the whole $\mathbb{R}^m$) of the equation (14).

To solve the above-formulated problems we use the Tikhonov regularization method, i.e. instead of the equation (16), we consider the following operator equation:

$$(A^h)^T A^h q^{\alpha} + \alpha \cdot E\, q^{\alpha} = (A^h)^T f^{\delta}, \tag{17}$$

where $\alpha = \alpha(h,\delta) \in \mathbb{R}^1_{++}$ is the regularization parameter; E is the identity matrix.

An important issue that arises from the application of the Tikhonov regularization method is the problem of finding the optimal or quasi-optimal regularization parameter α for a given set of input data $\{A^h, f^{\delta}, h, \delta\}$. Thus, it is necessary to find a value α, which should not be underestimated, because, otherwise, the stability of the equation (17) and its solution differ negligibly from the original equation (16), and, at the same time, the overestimation α can considerably distort our equation, making the solution stable, but losing most the useful information. These considerations can be shown graphically as the Fig. 1. As it is seen from Fig. 1, we have to find a point of balance between the stability of the solutions q^{α} and its distance from the exact solution q^{exact}. That is the desired value.

In [10, 27, 28, 55-57, 73, 87, 101, 108] and other corresponding works, there are various methods are given in respect to the classification and precision for optimal or quasi-optimal finding of the regularization parameter $\alpha = \alpha(h,\delta)$, and error estimation $\left\| q^{\text{normal}} - q^{\alpha} \right\|_{\ell_2}$ of the regularized solution q^{α}. In all these approaches, the basic requirement / condition of the proximity of $\left\| q^{\text{normal}} - q^{\alpha} \right\|_{\ell_2}$ to

$$\left\| q^{\text{normal}} - q^{\alpha_{\text{exact}\_optimal}} \right\|_{\ell_2} = \min_{\alpha(h,\delta)>0} \left\| q^{\text{normal}} - q^{\alpha(h,\delta)} \right\|_{\ell_2}$$

by the asymptotics with $h \to 0$ and $\delta \to 0$, but not with finite $h \ge 0$ and $\delta > 0$.

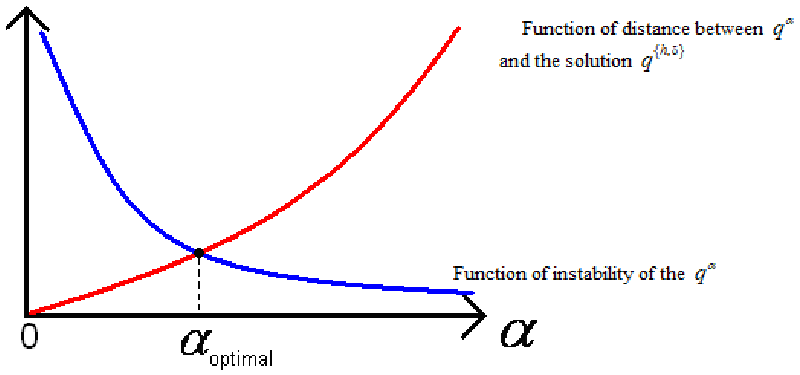

Fig. 1. Graphical representation of solution optimality concept

In other words, the traditional approaches with finite h and δ give quite good results only for the case of model problems, specially found for demonstration of the abilities of one or another method of finding the optimal and / or quasi-optimal regularization parameter.

Besides, another important point is that, applying the method of the Tikhonov regularization, we obtain the solution of the equation (17) instead of the solution of the equation (16), i.e. the original equation is the equation

$$\bar{A}^h q^{\alpha} = \bar{f}^{\{\delta,h\}}, \tag{18}$$

where $\bar{A}^h = (A^h)^T A^h$; $\bar{f}^{\{\delta,h\}} = (A^h)^T f^{\delta}$, which is the right-hand side f^{δ} of the initial equation (16), does not fit the classic method of the Tikhonov regularization in the direct form, however, in the various combinations of the Residual method, including the Generalized Residual principle, (for instance, see [73]), the error δ of the right-hand side is used f^{δ}, not the error of the right-hand side $\bar{f}^{\{\delta,h\}}$, which, as it could be seen from (18), depends not only on δ, but also on h. Therefore, random errors in f^{δ} can be smoothed quite well and, therefore, the relative error $\bar{f}^{\{\delta,h\}}$, which, as it was just noticed, in the classic method of the Tikhonov regularization, is not considered in any way, can significantly differ (even by several orders) from the relative error f^{δ}, which, as it was shown above, is the only one which is taken into account in the classic Tikhonov regularization method.

3.2.2. The proposed approach to solve the problem (18)

Now we consider an approach that takes into account the dependence of the right-hand side, not only on δ, but also on h, proposed in [57]. In other words, unlike the classical Tikhonov regularization method, the proposed approach is the change of the sequence of regularizing operator $\alpha \cdot E$ and the matrix $(A^h)^*$, which creates the fundamental difference in methodology for selecting regularization parameter α. According to the Generalized Residual principle (for instance, see [57, 101, 108]), the regularization parameter $\alpha = \alpha(h,\delta) \in \mathbb{R}^1_{++}$ is the root of the following equation:

$$\left\|A^h q^\alpha - f^\delta\right\|_{\ell_2(F)} = \left(\delta + h\cdot\left\|q^\alpha\right\|_{\ell_2(Q)}\right)^2 + \left(\inf_{q\in Q}\left\|A^h q - f^\delta\right\|_{\ell_2(F)}\right)^2, \qquad (19)$$

where $\inf_{q\in Q}\left\|A^h q - f^\delta\right\|_{\ell_2(F)}$ is a measure of the incompatibility of the original problem (15), (16). It has to be shown that, in the suggested approach, the measure of incompatibility of the equation (18), which is equivalent to $\bar{A}^h q^\alpha = \bar{f}^{\{\delta,h\}}$, is equal to zero, i.e. $\inf_{q\in Q}\left\|A^h q - f^\delta\right\|_{\ell_2(F)} = 0.$ That let us formulate and prove two following statements:

Statement 1. The equality $\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)} = \alpha\cdot\left\|q^\alpha\right\|_{\ell_2(Q)}$ is true.

Proof of the Statement 1. From the equation (17) it is obvious that $(A^h)^T A^h q^\alpha - (A^h)^T f^\delta = -\alpha\cdot E\, q^\alpha$, from which directly follows $\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)} = \alpha\cdot\left\|q^\alpha\right\|_{\ell_2(Q)}$. Statement 1 is proved.

Statement 2. $\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)}$ is a continuous and monotonously nondecreasing function in respect to the regularization parameter α.

Proof of the Statement 2. Since $\left\|q^\alpha\right\|$ is a continuous function (see [101]), then it is obvious that, after being multiplied by α, the continuous function $\alpha\cdot\left\|q^\alpha\right\|$ will be obtained. In [95], it is shown that

$$\left\|q^\alpha\right\|_{\ell_2(Q)} \le \alpha^{-\frac{1}{2}}\cdot\left\|f^\delta\right\|_{\ell_2(F)}. \qquad (20)$$

Therefore, $\lim_{\alpha\to 0^+}\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)} \le \lim_{\alpha\to 0^+}\sqrt{\alpha}\left\|f^\delta\right\|_{\ell_2(F)} = 0,$ i.e.

$$\lim_{\alpha\to 0^+}\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)} = 0. \qquad (21)$$

Besides that, using the fact that the norm is not negative, and also using inequality (20), it could be shown that $0 \le \lim_{\alpha\to+\infty}\left\|q^\alpha\right\|_{\ell_2(Q)} \le \left\|f^\delta\right\|_{\ell_2(F)}\cdot\lim_{\alpha\to+\infty}\alpha^{-\frac{1}{2}} = 0,$ hence, it follows that $\lim_{\alpha\to+\infty}\left\|q^\alpha\right\|_{\ell_2(Q)} = 0.$ Thus, we can write

$$\lim_{\alpha\to+\infty}\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)} = \left\|(A^h)^T f^\delta\right\|_{\ell_2(F)}.$$

As a result, we obtain

$$\left.\begin{aligned}&\lim_{\alpha\to 0^+}\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)} = 0,\\ &\lim_{\alpha\to+\infty}\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)} = \left\|(A^h)^T f^\delta\right\|_{\ell_2(F)}.\end{aligned}\right\} \qquad (22)$$

Besides limit expressions (22), we also know that $\left\|q^\alpha\right\|_{\ell_2(Q)}$ is a monotonously nonincreasing function in respect to the regularization parameter α (see [101]) and, therefore, we may claim that $\left\|(A^h)^T A^h q^\alpha - (A^h)^T f^\delta\right\|_{\ell_2(F)}$ is monotonous and nondecreasing function inrespect to the regularization parameter α. Statement 2 is proved.

Further, using these two proved statements, let us come back to the proof of the required incompatibility measure of equation (18):

$$\inf_{q\in Q}\left\|A^h q - f^{\delta}\right\|_{\ell_2(F)} = \lim_{\alpha\to 0^+}\left\|(A^h)^T A^h q^{\alpha} - (A^h)^T f^{\delta}\right\|_{\ell_2(F)} = 0,$$

i.e. we proved that incompatibility measure of the equation $\overline{A}^h q^{\alpha} = \overline{f}^{\{\delta,h\}}$ is equal to zero: $\inf_{q\in Q}\left\|A^h q - f^{\delta}\right\|_{\ell_2(F)}$. As it was shown in the work [56] (as well as see [57, 55]), the regularization parameter $\alpha = \alpha(h,\delta)\in \mathbb{R}^1_{++}$ is the root of the equation

$$\alpha^{\beta}\cdot\left\|(A^h)^T A^h q^{\alpha} - (A^h)^T f^{\delta}\right\|_{\ell_2(F)} = \gamma\cdot\left\|A^h\right\|_{\ell_2}\cdot\left(\delta + h\cdot\left\|z^{\alpha}\right\|_{\ell_2(Q)}\right), \tag{23}$$

where $\beta \geq 0$ and $\gamma > 0$ are some constants.

Using the statement 1, equation (23) could also be written as

$$\alpha^{\beta+1}\cdot\left\|q^{\alpha}\right\|_{\ell_2(Q)} = \gamma\cdot\left\|A^h\right\|_{\ell_2}\cdot\left(\delta + h\cdot\left\|q^{\alpha}\right\|_{\ell_2(Q)}\right). \tag{24}$$

Let us notice once more, that parameter α entries in equation (24) not only as left side multiplier, but it is also implicitly present in the q^{α}.

Below, there are four approaches of finding the regularization parameter α, basing on various assumptions and estimations, being considered in more details.

<u>The first approach of finding the regularization parameter</u>. The basis of the first approach mentioned is the assumption made earlier: for the equation (24) it is enough to use the original norm of the deviation of the operator and of the right-hand side, i.e. h and δ, of the equation (16) rather than the equation (18). This assumption gives us underestimated h and δ, so it also leads to an underestimated value $\alpha = \alpha(h,\delta)$, and, thus, does not move away from the original solution of (16) as well as makes it possible to find the solution without additional calculations connected to h and δ for the equation (18). On the other hand, a low value α gives smaller effect from use of the regularization method.

<u>The second approach of finding the regularization parameter</u>. Let us consider a different approach proposed in [113]. In this approach, the factor on the right-hand side of equation (24), namely $\Sigma_{Old}(\alpha) \overset{def}{\equiv} \left\|A^h\right\|\cdot\left(\delta + h\cdot\left\|q^{\alpha}\right\|_{\ell_2(Q)}\right)$, will transform and find a lower bound, thus, we obtain, as in the previous method, not an overestimated value α and not an overestimated residual $\left\|q^{\text{normal}} - q^{\alpha}\right\|_{\ell_2(Q)}$. Let us show that the expression, which was proposed in [113], namely $\Sigma_{New}(\alpha) \overset{def}{\equiv} \sup_{h\geq 0,\delta>0}\left\|\overline{f} - \overline{f}^{\{\delta,h\}}\right\|_{\ell_2(F)} + \left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot\sup_{h\geq 0}\left\|\overline{A} - \overline{A}^h\right\|_{\ell_2}$ does not exceed the original value $\Sigma_{Old}(\alpha)$, where

$$\overline{A} = (A^{\text{exact}})^T A^{\text{exact}},\ \ \overline{f} \overset{def}{\equiv} (A^{\text{exact}})^* f^{\text{exact}};\ \overline{A}^h = (A^h)^T A^h;\ \overline{f}^{\{\delta,h\}} = (A^h)^T f^{\delta}.$$

Indeed,

$$\Sigma_{New}(\alpha) = \sup_{h\geq 0,\delta>0}\left\|(A^{\text{exact}})^T f - (A^h)^T f^{\delta}\right\|_{\ell_2(Q)}$$

$$+\left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot\sup_{h\geq 0}\left\|(A^{\text{exact}})^T A^{\text{exact}}-(A^h)^T A^h\right\|_{\ell_2}\leq\sup_{h\geq 0,\delta>0}\left\|(A^h)^T\left(f-f^{\delta}\right)\right\|_{\ell_2(F)}$$

$$+\left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot\sup_{h\geq 0}\left\|(A^h)^T\left(A^{\text{exact}}-A^h\right)\right\|_{\ell_2}\leq\left\|(A^h)^T\right\|_{\ell_2}\cdot\delta+\left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot\left\|(A^h)^T\right\|_{\ell_2}\cdot h$$

$$+\left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot\sup_{h\geq 0}\left\{\left\|(A^h)^T\right\|_{\ell_2}\cdot\left\|A^{\text{exact}}-A^h\right\|_{\ell_2}\right\}\leq\left\|(A^h)^T\right\|_{\ell_2}\cdot\delta+\left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot\left\|(A^h)^T\right\|_{\ell_2}\cdot h$$

$$=\left\|A^h\right\|_{\ell_2}\cdot\left(\delta+\left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot h\right)=\Sigma_{Old}\left(\alpha\right).$$

Thus, if we use the resulting "new" sum $\Sigma_{New}\left(\alpha\right)$ instead of the "old" $\Sigma_{Old}\left(\alpha\right)$, then, instead of the equation (24) we have the following one:

$$\left\|q^{\alpha}\right\|_{\ell_2(Q)}\cdot\left(\alpha^{\beta+1}-\gamma\cdot\sup_{h\geq 0}\left\|\bar{A}-\bar{A}^h\right\|_{\ell_2}\right)=\gamma\cdot\sup_{h\geq 0,\delta>0}\left\|\bar{f}-\bar{f}^{\{\delta,h\}}\right\|_{\ell_2(F)}. \qquad (25)$$

So, we have $\Sigma_{New}\left(\alpha\right)\leq\Sigma_{Old}\left(\alpha\right)$ for $\forall\alpha\in\mathbb{R}^1_{++}$, and, therefore, as it has been mentioned earlier, the root $\alpha^{\text{optimal}}=\alpha^{\text{root}}\in\mathbb{R}^1_{++}$ of equation (25) and the residual $\left\|z^{\text{normal}}-z^{\alpha^{\text{optimal}}}\right\|_{\ell_2(Q)}$ are not overestimated. If we assume that there are conditions, restrictions

$$\left.\begin{array}{ll}\dfrac{\sup\limits_{h\geq 0,\delta>0}\left\|\bar{f}-f^{\{\delta,h\}}\right\|_{\ell_2(F)}}{\left\|f^{\{\delta,h\}}\right\|_{\ell_2(F)}}<\gamma^{-1} & if\quad\beta=0,\\ \left\|f^{\{\delta,h\}}\right\|_{\ell_2(F)}\neq 0 & if\quad\beta>0,\end{array}\right\} \qquad (26)$$

then the solution of the equation (25) will give rise to a new regularization operator with $\alpha^{\text{root}}\in\mathbb{R}^1_{++}$. In other words, conditions-restrictions (26) are important sufficient conditions to receive the regularizing operator, using the above-described method, and ignoring them does not guarantee that the approach necessarily generates the desired regularizing method.

<u>The third approach of finding the regularization parameter</u>. Assume that the residual of the normal and regularized solutions, as indicated in [101], has the following order

$$\left\|\bar{A}^h q^{\alpha}-\bar{f}^{\{\delta,h\}}\right\|_{\ell_2(F)}=O(\alpha^{-1}\left(h+\delta\right)^2). \qquad (27)$$

Let us note that, instead of the Euclidean norm $\left\|\cdot\right\|_{\ell_2}$, we can use the energy norm (also called the spectral norm): $\left\|v\right\|_B=\sqrt{(Bv,v)}$, $v\in\mathbb{R}^k$. Then, we find the energy norm for the equation (27) generated by the operator $\left(\bar{A}^h\right)^T$:

$$\left\|\bar{A}^h q^{\alpha}-\bar{f}^{\{\delta,h\}}\right\|^2_{\left(\bar{A}^h\right)^T}=O\left(\alpha^{-2}\cdot\left(h+\delta\right)^4\right).$$

So, now instead of the equation (24), which is used to find the regularization parameter α, we can consider the equation

$$\left\|A^h q^{\alpha}-f^{\delta}\right\|^2_{\ell_2(F)}=C\cdot\left\|A^h\right\|^{-3}_{\ell_2}\cdot\left(\delta+h\cdot\left\|q^{\alpha}\right\|_{\ell_2(Q)}\right)^2, \qquad (28)$$

where $\forall C \in \left[10^{-5};10^{-2}\right]$.

The fourth approach of finding the regularization parameter. In this approach, in order to find the regularizing parameter α there is a new equation proposed (see [5])

$$\left\|A^h q^\alpha - f^\delta\right\|_{\ell_2(F)}^2 = \frac{\left(\delta + h \cdot \left\|q^\alpha\right\|_{\ell_2(Q)}\right)^2}{\Phi\left(m_\varepsilon\right)}, \tag{29}$$

where

$$\begin{aligned}
&\Phi\left(m_\varepsilon\right) = 6 \cdot 10^{-6} + \left(11 - m_\varepsilon\right) \cdot \left[0.110999 + \left(1 - m_\varepsilon\right)\right.\\
&\times\left[-0.00179988 + \left(6 - m_\varepsilon\right) \cdot \left[-0.00521256 + \left(9 - m_\varepsilon\right)\right.\right.\\
&\times\left[-0.718663 + \left(3 - m_\varepsilon\right) \cdot \left[-0.102426 + \left(10 - m_\varepsilon\right)\right.\right.\\
&\times\left[-0.841212 + \left(2 - m_\varepsilon\right) \cdot \left[-0.163198 + \left(7 - m_\varepsilon\right)\right.\right.\\
&\times\left[-0.071326 + \left(4 - m_\varepsilon\right) \cdot \left[-0.0112364 - 0.00310896 \cdot \left(8 - m_\varepsilon\right)\right]\right]\left.\left.\left.\left.\left.\left.\left.\right]\right]\right]\right]\right]\right]\right].
\end{aligned}$$

The function $\Phi\left(m_\varepsilon\right)$ of equation (29) has been obtained on the basis of the sourcewise principle for the exact solution (which is unknown); the possibility of using (29) was examined in the article [41].

For the case when the value of m_ε is unknown, it is proposed to find a lower bound estimate from the equation error norm, namely

$$\left\|\Delta\delta\right\|_{\ell_2} = \sqrt{m \cdot \left(10^{-m_\varepsilon}\right)^2} = \delta, \tag{30}$$

where $m = \dim\left(f^\delta\right)$. Transforming the equation (30) we obtain the desired estimate for m_ε

$$m_\varepsilon = -\log_{10}\left(\frac{\delta}{\sqrt{m}}\right). \tag{31}$$

Finally, let us note that the fourth approach (i.e. formulas (29), (31)) is used for solving the finite-dimensional operator equations (5) and (6) to find the sought-for values of $Z = \left\{z_k \in \mathbb{R}_{++}^1\right\}_{k=\overline{1,K}}$ and $W(k) = \left\{w(k)_i \in \mathbb{R}_{++}^1\right\}_{i=\overline{1,I}}$, $\forall k = \overline{1,K}$, respectively.

3.2.3. General principle of algorithms of finding the normal solution (solution with minimal norm) of the first kind operator equation with completely continuous operator

Another approach, which is considered below, is focused on the solution of the equation (16) through optimization problem:

$$\min_{q \in Q}\left\|Aq - f\right\|_{\ell_2(F)}. \tag{32}$$

Let us designate through $\{R\}$ the class of all the positive definite self-adjoint operators Υ, such that the quadric form $\|Az\|^2_{\ell_2(F)} + (\Upsilon z, z)_Q$ is not less than $\theta^2 \cdot \|q\|^2_{\ell_2(Q)}$, in other words $\|Aq\|^2_{\ell_2(F)} + (\Upsilon q, q)_Q \geq \beta^2 \|q\|^2_Z$, where $\theta = \theta(A, \Upsilon) \in \mathbb{R}^1_{++}$ is a constant, not dependent on $q \in D(\Upsilon)$. Let us introduce the functional $M[\![\Upsilon, q, f]\!] = \|Aq - f\|^2_{\ell_2(F)} + (\Upsilon q, q)_Q, q \in D(\Upsilon)$. Then the element $z^{\text{exact}} \in D(\Upsilon)$, which minimizes the functional $M[\![\Upsilon, q, f]\!]$, satisfies the equation $(\Upsilon + A^* A)q = A^* f$, having the unique solution $q = R(\Upsilon) f$, where $R(\Upsilon) = (\Upsilon + A^* A)^{-1} A^*$; A^* is the operator, which is conjugate to the operator A $\left(A^* = A^T \text{ in the case of } A \in \mathbb{R}^{m\times n}\right)$. Then the element f is given approximately, in other words, it is assumed that $\tilde{f} = f + \tau$, where τ is a certain random process with the value in F, for which the probabilistic average is zero: $\mathrm{E}[\{\tau\}] = 0$. Let us designate

$$\Delta^2[\Upsilon, q(f), \tau] = \mathrm{E}\left[\|R(\Upsilon)\tilde{q} - q(f)\|^2_Q\right]$$

and

$$\tilde{\Delta}^2[\Upsilon, \{q(f)\}, \{\tau\}] = \sup_{q(f)\in\{q(f)\}, \tau\in\{\tau\}} \Delta^2[\Upsilon, q(f), \tau],$$

where $\{q(f)\}$ is the class of the admissible solutions of the problem (32), and it is the class of the admissible perturbations of the optimization problem (32). The optimal regularization is the operator $\Upsilon^{\text{optimal}} = \arg\inf_{\Upsilon\in\{\Upsilon\}} \tilde{\Delta}^2[\Upsilon, \{q(f)\}, \{\tau\}]$, i.e. $\Upsilon^{\text{optimal}} \in \{\Upsilon\}$ is determined as the solution of the problem

$$\inf_{\Upsilon\in\{\Upsilon\}} \tilde{\Delta}^2[\Upsilon, \{q(f)\}, \{\tau\}]. \tag{33}$$

If the solution of the extremum problem (33) exists, in this case the element $z^{\textbf{optimal}} \in R^{\textbf{optimal}}(\Upsilon)\tilde{f}$ is called the $(\Upsilon, \{q(f)\}, \{\tau\})$-optimal regularized solution of the problem (32). In this case, the value

$$\tilde{\Delta}^{\text{optimal}}[\Upsilon^{\text{optimal}}, \{q(f)\}, \{\tau\}] = \sup_{\Upsilon\in\{\Upsilon\}} \tilde{\Delta}[\Upsilon, \{q(f)\}, \{\tau\}]$$

is error of the $(\Upsilon, \{q(f)\}, \{\tau\})$-optimal regularized solution. In case when $\{\Upsilon\} = \{\Upsilon : \Upsilon = \alpha \cdot E\}$, where E is a unit operator, and $\alpha \in \mathbb{R}^1_{++}$ is a certain parameter (in general, it is unknown) called the parameter of regularization, and the extremum problem (33) is reduced to the determination of the optimal value of the regularized parameter $\alpha = \alpha^{\text{optimal}}$. Then we assume that:

- $q(f)$ is the solution for the problem (32) with the minimal admissible norm, in other words, we assume that $\{\Upsilon\} = \{\Upsilon : \Upsilon e_i = \alpha_i \;\; e_i, \forall i \in \mathbb{N}\}$, where $\{e_i\}_{i\in\mathbb{N}}$ is orthonormalized eigenvalues system of the self-adjoint operator A^*A, i.e. $A^*Ae_i = \omega_i \quad e_i,\ i\in\mathbb{N};\ \left\{\omega_i \in \mathbb{R}^1_{++}\right\}_{i\in\mathrm{N}} \downarrow;$

– $\tau \in \{\tau\}_\gamma = \left\{\tau_i : \mathrm{E}\left[\left\{\tau_i^2\right\}\right] = \gamma_i^2,\ 0<\gamma \ge \sup\limits_{i\in\mathbb{N}} \tau_i^2\right\}.$

Then, the $\left(\Upsilon, \{q(f)\}, \{\tau\}\right)$-optimal regularized solution can be expressed through the formula

$$q^{\text{optimal}} = \sum_{i=1}^{\infty} \frac{f_i^2 \cdot \left(f_i + \tau_i\right)}{\sqrt{\omega_i} \cdot \left(f_i^2 + \gamma_i^2\right)} e_i, \tag{34}$$

where

$$\left.\begin{aligned} &\left(\tilde{\Delta}^{\text{optimal}}\left[\Upsilon^{\text{optimal}}, \{q(f)\}, \{\tau\}\right]\right)^2 = \sum_{i=1}^{\infty} \frac{\gamma_i^2 \quad u_i^2}{\omega_i \quad \left(f_i^2+\gamma_i^2\right)}, \\ &\alpha_i^{\text{optimal}} = \frac{\omega_i \quad \gamma_i^2}{u_i^2},\ \forall i\in\mathbb{N}. \end{aligned}\right\} \tag{35}$$

From (34) and (35) it is possible to obtain the formula for the $\left(\Upsilon, \{q(f)\}, \{\tau\}_\gamma\right)$-optimal regularized solution $z_\gamma^{\text{optimal}}$:

$$z_\gamma^{\text{optimal}} = \sum_{i=1}^{\infty} \frac{f_i^2 \quad \left(f_i + \tau_i\right)}{\sqrt{\omega_i} \quad \left(f_i^2 + \gamma_i^2\right)} e_i, \tag{36}$$

where

$$\left.\begin{aligned} &\left(\tilde{\Delta}^{\text{optimal}}\left[\Upsilon^{\text{optimal}}, \{q(f)\}, \{\tau\}_\gamma\right]\right)^2 = \gamma^2 \quad \sum_{i=1}^{\infty} \frac{f_i^2}{\omega_i \quad \left(f_i^2+\gamma_i^2\right)}, \\ &\alpha_i^{\text{optimal}} = \gamma^2 \quad \frac{\omega_i}{f_i^2},\ \forall i\in\mathbb{N}. \end{aligned}\right\} \tag{37}$$

It is obvious that, in case of the finite-dimensionality of the operator $A:Q\to F,\ A\in\mathbb{R}^{m\times n},\ Q\subseteq\mathbb{R}^{n\times 1},\ F\subseteq\mathbb{R}^{m\times 1}$ of the original equation (16), the above-demonstrated formulas (34), (35) and (36), (37) stand valid, only if the lines in these formulas are exchanged by the corresponding sums. For instance, for this case the $\left(\Upsilon, \{q(f)\}, \{\tau\}_\gamma\right)$-optimal regularized solution $z_\gamma^{\text{optimal}}$ of the matrix equation (16) will be the regularized solution of the equation $\left(A^*A + \alpha\cdot E\right)q = A^*\left(f+\tau\right)$ for $\alpha = \alpha^{\text{optimal}} = \dfrac{\gamma^2}{c^2}$, and the corresponding error $\left(\tilde{\Delta}^{\text{optimal}}\left[\Upsilon^{\text{optimal}}, \{q(f)\}, \{\tau\}_\gamma\right]\right)^2$ will be the following (see [41]):

$$\left(\tilde{\Delta}^{\text{optimal}}\left[\Upsilon^{\text{optimal}}, \{q(f)\}, \{\tau\}_\gamma\right]\right)^2 = \gamma^2\cdot c^2 \cdot \sum_{i=1}^{m} \frac{1}{\gamma^2 + c^2\cdot\omega_i^2},$$

where $\{q(f)\}_m = \left\{q(f):\ q(f)\in Q,\ f_i^2 \le c^2,\ \forall i=\overline{1,m},\ c\equiv const.\right\}.$

Imposing these restrictions on the coefficients rate of decay $f_i,\ i=\overline{1,m}$ and the values $\gamma_i,\ i=\overline{1,m},$ it is possible to receive the exact values from the error $\tilde{\Delta}^{\text{optimal}}\left[\Upsilon^{\text{optimal}},\{q(f)\},\{\tau\}\right].$

3.2.4. Using the terms "solvability degree" and "correctness degree" for development of variational approach to stably solve the first kind finite-dimensional operator equation

Once again we consider the equation (16) with the a priori given additional information (15) and information about existence of solution.

So, let

$$B^h q = \overline{f}^{\{h,\delta\}}, \tag{38}$$

where $B^h=(A^h)^T A^h \in \mathbb{R}^{n\times n};\ \bar{f}^{\{h,\delta\}}=(A^h)^T f^{\delta} \in \mathbb{R}^{n\times 1}.$

Then, instead of original equation (16), we must consider (see [39]) the equation (38), where it is required to define the vector, which continuously depends on the initial data $\{A^h, h;\ f^{\delta}, \delta\}$ of equation (38), i.e. it has the property of stability (as for the Tikhonov meaning: (for instance, see [27, 28, 39, 101, 73]). Let us suppose the solution exists (in problems describing real processes or objects, that assumption is true due to existing system is diagnosed), then the solution can be shown through the regularizing operator $q^{\alpha}=R^{\alpha} f^{\delta}$. We will use the following concept ideas.

The solvability degree of the problem we define as

$$r_0=\frac{\delta}{\varepsilon(\delta)}=\frac{\|\Delta f\|_{\ell_2}}{\|\Delta q\|_{\ell_2}}. \tag{39}$$

Then the full solvability degree of the problem can be defined as

$$r_{full}=\inf_{\|\Delta f\|_{\ell_2}\le\delta} r_0=\inf_{\|\Delta f\|_{\ell_2}\le\delta}\frac{\|\Delta f\|_{\ell_2}}{\|\Delta q\|_{\ell_2}} \tag{40}$$

that for our problem means $r_c=\dfrac{1}{\|R_C\|_{\ell_2}}$.

The problem correctness degree d_0 we define in the following way:

$$\frac{1}{d_0}=\frac{\|R_C f-q\|_{\ell_2}}{\|q\|_{\ell_2}}=\frac{\left\|\left(R_C B^h-E\right)q\right\|_{\ell_2}}{\|q\|_{\ell_2}}, \tag{41}$$

where E is an identity matrix. Then the full correctness degree d_{full} can be defined as

$$\frac{1}{d_{full}}=\sup_{q\in F_C}\frac{1}{d_0}=\sup_{z\in F_C}\frac{\left\|\left(R_C B^h-E\right)q\right\|_{\ell_2}}{\|q\|_{\ell_2}}=\left\|\left(R_C A-E\right)\right\|_{\ell_2}, \tag{42}$$

where F_C is a correctness set of the operator R_C.

Using mentioned above notations we can formulate our problem as search of such operator R to have maximally high correctness degree, at the same time having given degree of solvability: $\sup\limits_{R_C} d_{full},\ r_C = const,$ or in other form

$$\begin{cases} \inf\limits_{R_C} \left\| \left(R_C B^h - E \right) \right\|_{\ell_2}, \\ \left\| R_C \right\|_{\ell_2} = C = const. \end{cases} \tag{43}$$

Problem (43) is a constrained optimization problem, and it can be reduced to the corresponding unconstrained optimization problem

$$\inf_{R_\alpha} \left\{ \left\| \left(R_\alpha B^h - E \right) \right\|_{\ell_2}^2 + \alpha \cdot \left\| R_\alpha \right\|_{\ell_2}^2 \right\}, \tag{44}$$

or in other form

$$\inf_{r_{j,i}^\alpha} \sum_{i=1}^n \sum_{j=1}^n \left\{ \left[\sum_{k=1}^n r_{j,k}^\alpha \cdot b_{k,i} - E_{j,i} \right]^2 + \alpha \cdot \left[r_{j,i}^\alpha \right]^2 \right\}, \tag{45}$$

where $E_{j,i} = \begin{cases} 1 \text{ if } \ j = i, \\ 0 \ \text{ otherwise} \end{cases}$ for $\forall \left(j = \overline{1,n};\ i = \overline{1,n} \right)$; $\left\{ r_{i,j} \right\} = R^\alpha \in \mathbb{R}^{n \times n}$; $\left\{ b_{i,j} \right\} = B^h \in \mathbb{R}^{n \times n}$. As it could be seen, regularization parameter α (that acts as an analogue of the Lagrange multiplier in the regular Lagrange function in the process of reducing the problem of constrained optimization to unconstrained) optimal value search problem is equal to search of the constant C.

Extremum problem (45) is equivalent to the following problem, which is extremum condition for (45):

$$\alpha \cdot r_{j,i}^\alpha + \sum_{l=1}^n r_{j,l}^\alpha \cdot \beta_{l,i} = b_{i,m},\ \forall \left(i = \overline{1,n};\ j = \overline{1,n} \right),$$

where $\beta_{l,i} = \sum\limits_{p=1}^n b_{l,p} \cdot b_{i,p}$.

As a result, we obtain the sequence of matrixes

$$\left\{ R^{\alpha_l} \right\}_{l=\overline{1,n}} = \left\{ \left\{ r_{i,j}^{\alpha_l} \right\}_{i,j=\overline{1,n}} \right\}_{l=\overline{1,n}} \in \mathbb{R}^{n \times n}.$$

A quasi-optimal value of the parameter α is selected by one of the following criteria (the validity of choice is proven in [23, 106, 111], where applicability of the selection criteria for solving various linear finite or infinite-dimensional operator equations of the first kind is illustrated):

- $\alpha^{\text{optimal}} = \arg\inf\limits_{\alpha} \left\{ \dfrac{1 + \varepsilon \cdot d_{full}^\alpha}{r_{full}^\alpha \cdot \left(d_{full}^\alpha - 1 \right)} \right\}$ under the condition
- $1 < d_{full}^\alpha = \dfrac{1}{\left\| R^\alpha B^h - E \right\|_{\ell_2}}$, where $r_{full}^\alpha = \left\| \left(B^h \right)^{-1} \right\|_{\ell_2}^{-1}$; E is unit operator / matrix;

ε is the maximum ratio error of the right-hand side of equation (38);

- $\alpha^{\text{optimal}} = \arg\inf_{\alpha} \sup_{\left\| \overline{f}^{\{\delta,h\}} - f^{\text{exact}} \right\|_{\ell_2} \le \delta} \left\{ \dfrac{\left\| B^h R^{\alpha} \overline{f}^{\{\delta,h\}} + f^{\{h,\delta\}} - R^{\alpha} \overline{f}^{\{\delta,h\}} \right\|_{\ell_2}^2}{\left\| B^h R^{\alpha} \overline{f}^{\{\delta,h\}} - f^{\{h,\delta\}} \right\|_{\ell_2}^2} \right\};$
- $\alpha^{\text{optimal}} = \arg\inf_{\alpha} \left\{ \dfrac{\varepsilon \cdot d_{full}^{\alpha} + 1}{r_{full}^{\alpha} \cdot d_{full}^{\alpha} \cdot \left\| R^{\alpha} B^h \right\|_{\ell_2}} \right\};$
- $\alpha^{\text{optimal}} = \arg\inf_{\alpha} \left\{ \dfrac{1}{\left(r_{full}^{\alpha} \right)^2 \cdot \left\| R^{\alpha} f^{\{h,\delta\}} \right\|_{\ell_2}} \right\}.$

Now, having α^{optimal} using the regularized solution of original problem (16), it is assumed that the vector $z^{\alpha^{\text{optimal}}} = R^{\alpha^{\text{optimal}}} \overline{f}^{\{\delta,h\}}$ and, thus, the regularized inverse operator / matrix $R^{\alpha^{\text{optimal}}} = \left(A^h \right)^{-1}$ is defined. It should be emphasized that these criteria are not equivalents, i.e. a choice of a criterion influences the accuracy of the required solution.

4. Computing experiment and the obtained results on the investigation of the economic and financial attractiveness of the Central and Eastern Europe countries

4.1. The choice of key socio-political, macroeconomic and financial parameters

In the current section, we describe a calculational experiment, where:

- 16 countries of the Central and Eastern Europe have beens selected from the study: Albania (AL), Bosnia and Herzegovina (BA), Bulgaria (BG), Croatia (HR), Czech Republic (CZ), Estonia (EE), Hungary (HU), Latvia (LV), Lithuania (LT), Macedonia (MK), Montenegro (ME), Poland (PL), Romania (RO), Serbia (RS), Slovakia (SK), and Slovenia (SL);
- as the studied time period we have selected [2010, 2016];
- 43 following key social political indicators, macroecnomic figures, and business environment factors have been selected as the parameters / indices (see Tab. 4); let us note that some of the listed parameters / indices have universally recognized abbreviations, for instance, IMF – International Monetary Fund; LFS – Labour force survey; GDP – Gross Domestic Product; PPP – Purchasing Power Parity; FDI – Foreign Direct Investment; EDB – Economic Development Board; CF – Capital Flight; HDR – Human Development Index.

Table 4. Socio-political, macroeconomic and financial parameters / indices Parameters / indices that are key in the investigation of the economic and financial attractiveness of the country

Conventional alpha-numeric designation	Parameter / index
P1	Population, in thousand
P2	Population of largest cities with agglomerations, in thousand
P3	Largest agglomerations, population $\geq$ 200 000
P4	Urbanization, in thousand
P5	Number of adults 20-year-old or above, in thousand
P6	Total wealth, in billion EUR
P7	Wealth per adult, in EUR
P8	Financial wealth per adult, in EUR
P9	Non-financial wealth per adult, in EUR
P10	Debt per adult, in EUR
P11	Median wealth per adult, in EUR
P12	Wealth range of adults: $\leq$ 10 000 EUR, in %
P13	Wealth range of adults: 10 000 ÷ 100 000 EUR, in %
P14	Wealth range of adults: over 100 000 ÷ 1 000 000 EUR, in %
P15	Wealth range of adults: $<$ 1 000 000 EUR, in %
P16	Gini index
P17	Total Gross Domestic Product, in billion EUR
P18	Gross Domestic Product real change, in %
P19	Gross Domestic Product per capita, in EUR at Purchasing Power Parity
P20	Unemployment rate, average, in %
P21	Average gross monthly wages, in EUR
P22	Consumer prices per annum, in %
P23	Fiscal balance of Gross Domestic Product, in %
P24	Public debt of Gross Domestic Product, in %
P25	Current account of Gross Domestic Product, in %
P26	Foreign Direct Investment inflow, in million EUR
P27	Gross external debt of Gross Domestic Product, in %
P28	Exchange Rate stability
P29	Ease of Doing Business Ranking, in position
P30	Foreign Direct Investment / Paying Taxes
P31	Foreign Direct Investment / Tax Burden, in %
P32	Foreign Direct Investment / Trading Across Borders, in position
P33	Corruption Perception index
P34	Credit Rating (average, 0÷100)
P35	Net International Investment Position, in billion EUR
P36	Foreign Direct Investment balance, in billion EUR
P37	Portfolio Investment balance, in billion EUR
P38	Other investment balance, in billion EUR
P39	Capital Flight calculated by the World Bank method, in million EUR
P40	Quality of Life Index
P41	Human Development Index
P42	Offshore concentration defined on indirect indications
P43	Ratio of financial and non-financial wealth

The parameters / indices P15, P16, P20, P28, P29, P35, unlike from the other 37 parameters are such that the bigger the value of each for these 6 parameters is the lesser attractiveness of a country for exports of banking and other luxourious services becomes: in the subsection 2.1, such parameter had been conditionally labeled as inverse parameter / index, so we have claimed that for such parameters / indices it is necessary to homological scaling, which lets get rid of the indicated inverse proportionality.

Values of all the 43 parameters / indices given in the Tab. 4 for the period of 7 years (2010-2016) in the 16 studied countries have been taken from the listed resources [105]; moreover, these values were also compared with the values from the relevant national sources from these 16 countries, if they consisted of such values. Here, we should note that for some parameters / indices their values in the transnational organizations'(The World Bank, The Eurostat of the European Commission, The Economist Intelligence Unit, The Vienna Institute for International Economic Studies, etc.) and in national data resources' sources do differ: in such kind of situations, we have opted to pick the data given in the transnational organizations' datasets. Finally, we should also point that Kosovo, which is officially considered an Eastern European country, is not included in the current paper as a country, whose economic-financial attractiveness / potential is studied. The absence of Kosovo is tied to the fact that multiple international organizations' either do not have relevant data on Kosovo, or such data is absent for selected periods of time, or that data differs from its analogues from the Kosovar sources dramatically.

Before we apply the developed methods and algorithms to the mathematical model (1)-(4), the values of all the 43 parameters had been subejected to the prestarting procedure, whose steps and stages are mentioned in detail in the subsection 2.1.

4.2. The results obtained by the realization of the mathematical model: the economic-financial attractiveness of countries, the significance coefficient of parameters, the "degree of favorability" of the years, the coefficient of relative strengthening of the importance of parameters

Before we disclose the obtained results, webriefly address the realization of the suggested mathematical model (1)-(4) through the developed methods, which are described in details in the section 3.

Stage 1. For each of the $K=43$ parameters / indices of equation (1)-(2) mathematical model (1)-(4) represents a system of the $I+J=23$ interrelated equations in respect towards the same amount of the unknowns $\{x(k)_1,\ldots,x(k)_{J=16};y(k)_1,\ldots,y(k)_{I=7}\}$, where $x(k)_j$ stands for the sought-for economic attractiveness / potentialof the j-th $j=\overline{1,(\text{J=16})}$ country by the k-th $k=\overline{1,(K=43)}$ parameter / index; $y(k)_i$ $i=\overline{1,(\text{I=7})}$ stands for the sought-for "degree of favourability" of the i-th $\left(i=\overline{1,(I=7)}\right)$ year by the k-th $k=\overline{1,(K=43)}$ parameter / index. Each of these $K=43$ systems of the interrelated equations had been solved through the iterative process, where

$$\left\|Y(k)\right\|_{\infty}=I\cdot J\cdot K=4816,\ \forall k=\overline{1,(K=43)},\ \lambda=J=16,\ \mu=0.5.$$

Stage 2. For each of the $K=43$ parameters / indices equation (3) of the mathematical model (1)-(4) represented an overdetermined system of linear algebraic equations $J=16$ equations in relation to the $I=7$ unknowns $\{w(k)_1,\ldots,w(k)_{I=7}\}$, where $w(k)_i$ for the sought-for significance coefficient of the k-th $k=\overline{1,(K=43)}$ parameter / index in the i-th $(i=\overline{1,(I=7)})$ year. Each of these $K=43$ systems had been solved by the Tikhonov regularization method, where, instead of the classical one, we solve modified equation (17) with the regularization parameter found according to the formulas (29), (30). As a result of application of that method, we find values of the sought-for variables $\{w(k)_1,\ldots,w(k)_{I=7}\}$ with an accuracy of $\varepsilon=10^{-9}$. The application of the Tikhonov regularization method guarantees the stability of the found solution with respect to small errors in the source data.

Stage 3. Equation (4) of mathematical model (1)-(4) represents an underdetermined system of linear algebraic equations with $J=16$ equations in relation to the $K=43$ unknowns $\{z_1,\ldots,z_{K=43}\}$, where z_k $(k=\overline{1,(K=43)})$ stands for the coefficient of relative strengthening of importance of the k-th $(k=\overline{1,(K=43)})$ parameter / index. That system was also solved through the Tikhonov regularization method, where again, instead of the classical Tikhonov equation, we solve the modified one (17) with the parameter of regularization found according to the formulas (29), (30). As a result of application of that method, we find the values of the sought-for variables $\{z_1,\ldots,z_{K=43}\}$ withanaccuracyof $\varepsilon=10^{-9}$. The application of the Tikhonov regularization method guarantees the stability of the found solution with respect to small errors in the source data.

The main results obtained by the computer implementation of the foregoing three stages of solving the investigated problem are presented below (see the Tab. 5-8). For computer realization of all three stages, a special user program was developed in the MathCAD environment (MathCAD 14, version 14.0.0.163). The developed program (in which there are more than 2000 code lines) uses, basically, only those built-in program modules of MathCAD 14, which relate to the arithmetic operations on vectors and matrices. All program modules, which implement:

– all three steps of the prestarting procedure (see the subsection 2.1) using the Standardization approach 1;
– the iterative process (12) for solving the $K=43$ systems of interrelated equations (1), (2);
– the Tikhonov regularization method (i.e. the equation (17) with the choice of the regularization parameter according to the formulas (29), (30)) for solving the $K+1=44$ systems (3) and (4),

were developed by the authors from scratch.

Unprocessed source data, i.e. the elements of the matrix P^{source} of the size $K\times(I\cdot J)=43\times112$, were prepared in MS Excel 2010 in the form of the Tab. 3. The importating of these data into the developed program module, which implements the preparatory procedure, was carried out only once using the MathCAD's dialogue component"Excel Setup Wizard".

Table 5. The economic-financial attractiveness / potential (overall assessment on all 43 parameters) of the Central and Eastern Europe countries from the standpoint of opportunity to establish exports of banking and other luxurious services to these countries

Country	Results	
	Rating	Economic-financial attractiveness / potential
Albania	16	1.014
Bosnia and Herzegovina	13	2.542
Bulgaria	10	3.954
Croatia	8	6.106
Czech Republic	2	8.140
Estonia	7	6.139
Hungary	3	7.781
Latvia	11	3.707
Lithuania	9	5.174
Macedonia	15	1.341
Montenegro	14	2.004
Poland	1	9.817
Romania	6	6.888
Serbia	12	3.511
Slovakia	5	7.206
Slovenia	4	7.698

Table 6. The "degree of favourability" of year (overall assessment on all 43 parameters

Ordered rating	Year	Degree of favourability (in logarithmic scale)
1	2016	3.999
2	2015	3.898
3	2012	3.679
4	2013	3.453
5	2014	3.099
6	2011	2.361
7	2010	1.330

Table 7. The Weighting coefficient of parameters (overall assessment on all 7years)

Parameter	Weighting coefficient		Parameter	Weighting coefficient	
	Nonnormalized	Nnormalized		Nonnormalized	Nnormalized
P1	0.74224	0.69109	P23	0.00111	0.15760
P2	0.65094	0.62235	P24	0.00200	0.15799
P3	1.46704	0.96202	P25	0.00200	0.15799

P4	0.48959	0.48983	P26	0.81312	0.73934
P5	0.49260	0.49234	P27	0.00008	0.15714
P6	0.77575	0.71451	P28	0.43011	0.44032
P7	0.74983	0.69649	P29	0.57075	0.55750
P8	0.42880	0.43924	P30	0.50998	0.50689
P9	0.39499	0.41158	P31	0.47132	0.47455
P10	0.08170	0.19681	P32	0.51227	0.50881
P11	0.08022	0.19602	P33	0.66215	0.63113
P12	0.51518	0.51124	P34	0.64066	0.61422
P13	0.56396	0.55189	P35	0.58821	0.57187
P14	1.03425	0.85605	P36	0.43984	0.44837
P15	1.16053	0.90076	P37	0.26002	0.30803
P16	0.65963	0.62916	P38	0.17903	0.25341
P17	0.61162	0.59094	P39	0.43681	0.44586
P18	0.53800	0.53031	P40	0.51725	0.51297
P19	0.51067	0.50747	P41	0.52673	0.52090
P20	0.43907	0.44773	P42	0.45977	0.46492
P21	0.65537	0.62583	P43	0.44394	0.45176
P22	0.06600	0.18863	Normalization formula:		

$$\left.\begin{aligned} w_i^{\text{normalized}} &= \frac{1}{1+e^{-\text{standard}_i}}, \\ \text{standard}_i &= \frac{w_i - m}{\sigma}, \end{aligned}\right\} \quad \forall i = \overline{1,(K=43)}$$

m is the mean value, σ is the standard deviation.

Table 8. The "coefficient of relative strengthening" of the impotance of parameter

Parameter	Coefficient of relative strengthening		Parameter	Coefficient of relative strengthening	
	Nonnormalized	**Nnormalized**		**Nonnormalized**	**Nnormalized**
P1	1.24032	0.28563	P23	0.91771	0.21243
P2	1.53311	0.36368	P24	0.93426	0.21582
P3	3.41900	0.85092	P25	0.96002	0.22119
P4	2.40110	0.62240	P26	2.15182	0.54849
P5	1.51404	0.35831	P27	0.96526	0.22229
P6	2.70321	0.70441	P28	3.27414	0.82707
P7	2.70141	0.70395	P29	1.47590	0.34769
P8	1.32160	0.30630	P30	2.31760	0.59818
P9	1.25610	0.28958	P31	2.32417	0.60008
P10	1.00241	0.23023	P32	1.11916	0.25643
P11	2.08112	0.52729	P33	1.95115	0.48767

P12	2.25651	0.58013	P34	2.97355	0.76821
P13	1.77033	0.43291	P35	1.22470	0.28176
P14	1.91570	0.47688	P36	3.00462	0.77490
P15	3.07551	0.78963	P37	1.78453	0.43717
P16	0.62132	0.15815	P38	2.12145	0.53953
P17	2.91450	0.75514	P39	2.10151	0.53349
P18	1.17562	0.26980	P40	3.09580	0.79371
P19	3.15480	0.80525	P41	2.97547	0.76862
P20	2.93580	0.75992	P42	2.94291	0.76129
P21	0.36437	0.12072	P43	1.84461	0.45529
P22	1.82071	0.44807	Normalization formula:		

$$\left.\begin{aligned} w_i^{\text{normalized}} &= \frac{1}{1+e^{-\text{standard}_i}}, \\ \text{standard}_i &= \frac{w_i - m}{\sigma}, \end{aligned}\right\} \forall i = \overline{1,(K=43)}$$

m is the mean value, σ is the standard deviation.

5. Conclusion

In the current paper, we study evaluation of the economic-financial attractiveness / potential of the 16 Central and Eastern European countries by values of the 43 key social political indicators, macroeconomic figures, and factors of business environment over the period of 2010-2016.

We develop the mathematical model, which consists of three parts: the first one consists of 43 undefined systems of algebraic equations; the second one consists of 43 undefined systems of algebraic equations, whose input data are the output data from the first part; the third part consists of one redefined system of algrbraic equations, whose input data also are the output data from the first part of the model. We prove existence and uniqueness of solvability of the suggested mathematical model and expose that such solution cannot be found by direct methods.

In the paper, there is an iterative process being suggested; with the help of which we are solving the first part of the mathematical model. We expose the unconditional convergence of the suggested iterative process. Based on the Tikhonov regularization method, we develop and argue in favour of the modified regularization algorithm for solving operator equation of the first kind (not only with the finite operator), so, with the help of that algorithm, we solve the other two parts of the suggested mathematical model.

Moreover, in the current paper, we conduct the calculational experiment, which realised the developed model and methods. From the multiple obtained results of the conducted calculational experiment, in this work, mostly only the averaged (by years and parameters) results are included: economic-financial attractiveness of each country by the aggregate of all the parameters;.degree of favourability of each year by the aggregate of all the parameters; weighting coefficients of each parameter by the aggregate of all the years; coefficient of relative enforcement of importance of each parameter. Such importance nonaveraged results as weighting coefficients of each parameter by each year; economic-financial attractiveness of each country by each of the parameters; degree of favourability of each year by each parameter; degree of succession of each year by each parameter; etc. are not included in the present paper.

The averaged results of the calculational experiment presented below let detect some interesting circumstances:

- by the economic-financial attractiveness (from the standpoint of experts of banking and other luxourious services), Estonia with its only 1.3M population is scoring better than Croatia (4M people) and is near Romania (almost 20M inhabitants);
- despite some loud statements by politcians of various level, Bulgaria appears to be more attractive country for exports of banking and other luxourious services than Latvia, and the reason of that also cannot be tied to the fact that the population of Latvia is notably smaller (Latvian population exceeds the Estonian one by 0.6M people);
- by the economic-financial attractiveness, Slovenia with its 2M population is in front of Lithuania, whose population is 3M;
- by the economic-financial attractiveness, small countries like Slovenia and Slovakia with their summed population of 7.5M are behind Poland (38M), Czech Republic (over 9M), and Hungary (about 9M) only, chasing by small margin, etc.;
- from the standpoint of the economic-financial attractiveness, 2012 was a better year than 2010, 2011 and even 2013, 2014;
- from the standpoint of the economic-financial attractiveness, last 2 years (in relation to the ongoing 2017) were the best over the last 7 years' period, etc.

Finally, one wishes to note that the ideas and approaches suggested in this paper can be dessiminated also in other domains of human knowledge, where it is necessary to execute monitoring of objects, process, or phenomenon, and, on the basis of obtained data, make some scientifically substantiated conclusions, decisions, diagnoses, forecasts, etc.

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References

1. Aivazyan, S.A., Yenyukov, I.S., Meshalkin L.D. (1985). *Applied Statistics: Study of Relationships.* Moscow, RF: Finance and Statistics Publishing, 487 p.
2. Andersen, K., Pochet, Y. (2007). Coefficient strengthening: A tool for formulating mixed integer programs. Core Discussion Paper of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State. *Prime Minister's Office, Science Policy Programming*, 24, 1-29.
3. Andersen, K., Pochet, Y. (2010). Coefficient strengthening: a tool for reformulating mixed-integer programs. *Journal of Mathematical Programming*, *122*(1), 121-154.
4. Anderson, R. (2010). Systematic reviews of economic evaluations: utility or futility?. *Journal of Health Economics*, *19*(3), 350-364.
5. Andreyev, S.A., Guseynov, Sh.E. (2013). *Regularizing algorithms for diagnosing: Applied to gas turbine engines in operation*. Saarbrucken, Germany: LAP Publishing, 116 p.
6. Andreyev, S.A., Guseynov, Sh.E., Yunusov, S.M., Labendik, V.P. (2013). Application of Tikhonov's Regularization Method for Stable Finding the Diagnostic Matrix of GTE with a priori Given Accuracy, Part III: Development of Methods for the Determination of the Regularization Parameter.*Journal of Mathematical Applications in Science & Mechanics*,*14*, 121-126.
7. Antonanzas, F., Rodriguez-Ibeas, R., Juarez, C., Hutter, F., Lorente, R., Pinillos, M. (2009). Transferability indices for health economic evaluations: methods and applications.*Journal of Health Economics*, *18*(6), 629-643.
8. Asaul, A.N., Pasyada, N.I. (2008). *Investment Attractiveness of the Region, Saint-Petersburg.* Saint-Petersburg State University of Architecture and Civil Engineering Press, 120 p.
9. Baker, D., Bridges, D., Hunter, R., Johnson, G., Krupa, J., Murphy, J., Sorenson, K. (2002). *Guidebook to Decision-Making Methods*. Washington, USA: U.S. Department of Energy, 44 p.
10. Bauer, F., Lukas, M.A. (2011). Comparing parameter choice methods for regularization of ill-posed problems. *Journal of Mathematics and Computers in Simulation, 81*(9), 1795-1841.
11. Birge, J. R., Linetsky, V. (Eds.). (2007). *Handbooks in operations research and management science: Financial engineering* (Vol. 15). Elsevier.
12. Blyum, E.A. (2013). Review of methodologies for assessing the investment potential of the region. *Journal of Young Scientist*, *54*(7), 137-141.
13. Borenstein, M., Hedges, L.V., Higgins, J.P.T., Rothstein, H.R. (2009). *Introduction to Meta-Analysis*. Padstow, UK: John Wiley & Sons Publishing, 421 p.
14. Central and Eastern Europe. Available on-line at: https://en.wikipedia.org/wiki/Central_and_Eastern_Europe [Accessed: November 11, 2017].
15. Chalenko, A. (2017). Methodology for determining the economic potential, Available on-line at:

http://kapital-rus.ru/articles/article/metodika_opredeleniya_ekonomicheskogo_potenciala/ [Accessed: November 11, 2017].
16. Chatterjee, K. (1981). Comparison of Arbitration Procedures: Models with Complete and Incomplete Information. *IEEE Transactions on Systems, Man, and Cybernetics, 11*(2), 101-109.
17. Cherkez, A.Ya. (1975). *Engineering Calculations of Gas Turbine Engines by Small Deviations' Method*. Moscow, RF: Engineering Industry Publishing, 380 p.
18. Cooke, R. M. (1991). Experts in Uncertainty Oxford University Press. *CookeExperts in Uncertainty*.
19. Cooper, N.J., Sutton, A.J., Ades, A.E., Paisley, S., Jones, D.R. (2007). Use of evidence in economic decision models: practical issues and methodological challenges. *Journal of Health Economics, 16*(12), 1277-1286.
20. Diskin, A., Koppel, M., Samet, D. (2011). Generalized Raiffa solutions. *Journal of Games and Economic Behavior, 73*(2), 452-458.
21. Dmitriev, V.I., Guseynov, Sh.E. (1995). On conformance of the resolving power and the solution detailedness in an inverse problems. *Herald of the Moscow State University, Series 15: Computational Mathematics and Cybernetics*, *1*, 17-25.
22. Draper, N.R., Smith, H. (1998). *Applied Regression Analysis.* New York, USA: John Wiley & Sons Publishing, 706 p.
23. Dunford, N., Schwartz, J.T. (1988). *Linear Operators. Part I: General Theory*. New York, USA: Wiley-Interscience, 858 p.
24. Durkin, J. (1994). *Expert Systems: Design and Development.*New York, USA: MacMillan Publishing, 800 p.
25. Edwards, W., Barron, F.H. (1994). SMARTS and SMARTER: improved simple methods for multiattribute utility measurement. *Journal of Organization behaviour and human processes,* 60, 306-325.
26. Ehrgott, M., Figueira, J.R., Greco, S. (2010). *Trends in Multiple Criteria Decision Analysis.* New York, USA: Springer-Verlag Publishing, 412 p.
27. Engl, H.W., Hanke, M., Neubauer A. (2000). *Regularization of Inverse Problems.* Amsterdam, The Netherlands: Springer-Verlag Publishing, 322 p.
28. Engl, H.W., Neubauer, A. (1985). Optimal discrepancy principles for the Tikhonov regularization of the first kind integral equations. *In: Constructive Methods for the Practical Treatment of Integral Equations (Eds. by Hammerlin G., Homann K.-H.), Basel-Boston-Stuttgart: Birkhouser Publishing*, 120-141.
29. Fishburn, P.C. (1967). Additive Utilities with Incomplete Product Set: Applications to Priorities and Assignments.*Journal of Operations Research,15*(3), 537-542.
30. Fishburn, P.C. (1970). *Utility Theory for Decision Making: Research Analysis Corporation.* New York, USA: John Wiley & Sons, 234 p.
31. Forman, E.H., Selly, M.A. (2001). *Decision by objectives: how to convince others that you are right*. World Scientific.
32. French, S. (2003). Modelling, making inferences and making decisions: the roles of sensitivity analysis. *Journal of TOP: An Official Journal of the Spanish Society of Statistics and Operations Research*, *11*(2), 229-251.
33. Geyskens, I., Krishnan, R., Steenkamp, J.-B.E.M., Cunha, P.V. (2009). A review and evaluation of meta-analysis practices in management research. *Journal of Management*, *35*(1), 393-419.
34. Gilovich, T., Griffin, D., Kahneman, D. (2002). *Heuristics and Biases: The Psychology of Intuitive Judgment.* Cambridge, UK: Cambridge University Press, 767p.

35. Greco, S., Ehrgott, M., Figueira, J.R. (2016). *Multiple Criteria Decision Analysis: State of the Art Surveys*. New York, USA: Springer-Verlag Publishing, 1347p.
36. Greshilov, A.A. (2014). *Mathematical methods of decision-making*. Moscow, RF: Bauman Moscow State Technical University Press, 647 p.
37. Gupta, S.C., Kapoor, V.K. (2014). *Fundamentals of Mathematical Statistics: A Modern Approach*, New Dehli, India: Sultan Chand & Sons Publishing, 1146p.
38. Guseynov, Sh.E., Aleksejeva, J.V., Andreyev, S.A. (2017). On one regularizing algorithm for comprehensive diagnosing of apparatus, engines and machinery. *Journal of Advanced Materials Research*, 254-257. Available on-line at: https://www.scientific.net/AMR.1117.254 [Accessed: November 11, 2017].
39. Guseynov, Sh.E., Okruzhnova, M. (2005). Choice of a quasi-optimal regularization parameter for the first kind operator equations. *Journal of Transport and Telecommunication, 6*(3), 471-486.
40. Guseynov, Sh.E., Urbah, A.I., Andreyev, S.A. (2015). On one Approach for Stable Estimate of Technical System Efficiency, In: *Proceedings of the 10th International Scientific and Practical Conference "Environment. Technology, Resources"*, Rezekne, Latvia, 3, 100-108, Available on-line at: http://dx.doi.org/10.17770/etr2015vol3.191 [Accessed: November 11, 2017].
41. Guseynov, Sh.E., Volodko, I. (2003). Convergence order of one regularization method. *Journal of Mathematical Modelling and Analysis, 8*(1), 25-32.
42. Guseynov, Sh.E., Yunusov, S.M. (2011). New regularizing approach to determining the influence coefficient matrix for gas-turbine engines.*In: Dynamical Systems, Differential Equations and Applications*,1, American Institute of Mathematical Sciences (AIMS), 614-623.
43. Guseynov, Sh.E., Yunusov, S.M., Andreyev, A., Labendik, V.P. (2013). Application of Tikhonov's Regularization Method for Stable Finding the Diagnostic Matrix of GTE with a priori Given Accuracy, Part II: Gas Path Analysis for Controlling the State of GTE Air Gas Channel with the Help of Tikhonov's Regularization Method. *Journal of Mathematical Applications in Science & Mechanics*, 14, 115-120.
44. Hammond, J.S., Keeney, R.L., Raiffa, H. (1999). *Smart Choices. A Practical Guide to Making Better Decisions.* Boston, USA: Harvard Business School Press, 243 p.
45. Himmelblau, D.M. (1970). *Process analysis by statistical methods.* New York, USA: John Wiley & Sons Publishing, 463 p.
46. Ibragimov, I.A., Hasminsky, R.Z. (1979). *Asymptotic theory of evaluation.* Moscow, RF: Science Publishing, 528 p.
47. Inozamtsev, A.A., Nikhamkin, M.A., Sandratsky, V.L. (2008). *Principles of Aero-Engines and Power Generation Gas Turbines Designing*. Volume V: Automatics and Control of Aero-Engines and Power Generation Gas Turbines, Moscow, RF: Engineering Industry Publishing, 200 p.
48. Jackson, P. (1998). *Introduction to Expert Systems*. Boston, USA: Addison-Wesley Longman Publishing, 526 p.
49. Juang, J.-N., Phan, M.Q. (2001). *Identification and Control of Mechanical Systems*. Cambridge, UK: Cambridge University Press, 334 p.
50. Kalai, E., Smorodinski, M. (1975). Other solutions to Nash's bargaining problem. *Journal of Econometrica, 43*(3), 513-518.
51. Kantorovich, L.V., Akilov, G.P. (1964). *Functional Analysis in Normed Spaces.* Oxford, UK: Pergamon Press, 771 p.
52. Karnopp, D.C., Margolis, D.L., Rosenberg, R.C. (2012). *System Dynamics: Modeling, Simulation, and Control of Mechatronic Systems.* Hoboken, USA: John Wiley & Sons, 636 p.

53. Keeney, R.L., Raiffa, H. (1993). *Decisions with Multiple Objectives: Preferences and Value Tradeoffs.* Cambridge, UK: Cambridge University Press, 592 p.
54. Klimova, N.I. (2003). *Investment attractiveness of the region*. Yekaterinburg, RF: Ural Division of the Russian Academy of Sciences Press, 276 p.
55. Kojdecki, M.A. (2001). Examples of Saturated Convergence Rates for Tikhonov Regularization. *BIT Numerical Mathematics*, *41*(5), 1059-1068.
56. Kojdecki, M.A. (1996). New criterion of regularization parameter choice in Tikhonov method. *Dissertation to obtaining the scientific degree PhD in Mathematics*, Warsaw, Poland: Institute of Mathematics of Polish Academy of Science Press, 127 p.
57. Kojdecki, M.A. (2000). New criterion of regularization parameter choice in Tikhonov method. *Bulletin of the Military University of Technology*, *49*(1), 47-126.
58. Korobov, B.V. (2005). Comparative analysis of methods for determination of weighting coefficients of influencing factors. *Journal of Sociology*, 20, 54-73.
59. Krantz, D.H., Luce, R.D., Suppes, P., Tverski, A., (2006). *Foundation of Measurement, Volume I: Additive and Polynomial Representations.* Mineola, USA: Dover Publishing, 607 p.
60. Krugman, P.R., Obstfeld, M., (2007). *International Economics: Theory & Policy*. Boston-San Francisco-New York, USA: Pearson Addison Wesley Publishing, 687 p.
61. Krugman, P.R., Wells, R. (2015). *Economics.* New York, USA: Worth Publishing, 1016 p.
62. Krugman, P.R., Wells, R., Graddy, K. (2011). *Essentials of Economics*. New York, USA: Worth Publishing, 604 p.
63. Larichev, O.I. (1985). *Decision Making as a Research Branch: Methodological Problems*. In: Methodological Problems of Systems Approach and Systems Analysis (Ed. by Gvishiani D.M.), Volume 2: Systems Research. Methodological Problems. Advances in Systems Research Series, Oxford, UK: Pergamon Press, 130-141.
64. Luce, R.D., Raiffa, H. (1957). *Games and Decisions: Introduction and Critical Survey*. New York, USA: John Wiley & Sons Publishing, 509 p.
65. Mahalik, N.P. (2003). *Fieldbus Technology: Industrial Network Standards for Real-Time Distributed Control*. Berlin, Germany: Springer-Verlag Publishing, 592 p.
66. Mahalik, N.P. (2003). *Mechatronics: Principles, Concepts and Applications.* New Delhi, India: Tata McGraw-Hill Publishing, 589 p.
67. Mechenov, A.S. (1976). Numerical solution of linear integral equations of the first kind by the regularization method, *Dissertation to obtaining the scientific degree PhD in Mathematics & Physics*, Moscow, USSR: Lomonosov Moscow State University Press, 138 p.
68. Mellor, F. (2010). *Negotiating Uncertainty*. Journal of Public Understanding of Science, *29*(1), 16-33.
69. Meyer, M.A., Booker, J.M. (1990). *Eliciting and analyzing expert judgment: a practical guide*. Washington, USA: U.S. Nuclear Regulatory Commission, 399 p.
70. Meyer, M.A., Booker, J.M. (2001). *Eliciting and Analyzing Expert Judgment: A Practical Guide*. Philadelphia, USA: Society for Industrial and Applied Mathematics, 459 p.
71. Montibeller, G., Gummer, H., Tumidei, D. (2006). Combining scenario planning and multi-criteria decision analysis in practice, *Journal of Multi-Criteria Decision Analysis*, *14*(1-3), 5-20.
72. Morgan, M.G., Fischhoff, B., Bostrom, A., Atman, C.J. (2001). *Risk Communication: A Mental Models Approach*. Cambridge, UK: Cambridge University Press, 351 p.
73. Morozov, V.A. (2001). *Methods of Solving Incorrectly Posed Problems*. New York-Berlin-Heidelberg: Springer-Verlag Publishing, 257 p.

74. Moulin, H. (1988). *Axioms of cooperative decision making*. Cambridge, UK: Cambridge University Press, 334 p.
75. Nash, J.F. (1950). The bargaining problem. *Journal of Econometrica, 18*(2), 155-162.
76. Natrins, A., Deikis, I., Guseynov, Sh.E., Leshinskis, K., Sarnovichs, A. (2015). *Fundamental Principles of Creation of Attractiveness Principles of Central and Eastern European Countries for Evaluation of Customer Segment and Financial Services Potential*. Riga, Latvia: Business and Financial Research Centre Press, 107 p.
77. Nemhauser, G.L., Rinnoy, Kan, A.H.G., Todd, M.J. (1989). *Handbooks in Operations Research and Management Science, Vol.1: Optimization*. Amsterdam, The Netherlands: Elsevier Publishing, 709 p.
78. Novikov, A.S., Paikin, A.G., Sirotin, N.N. (2007). *Control and Diagnostics of the Gas-Turbine Engine Performance.* Moscow, RF: Science Publishing, 469 p.
79. O'Hagan, A., Buck, C.E., Daneshkhah, A., Eiser, J.R., Garthwaite, P.H., Jenkinson, D.J., Oakley, J.E., Rakow, T. (2006). *Uncertain Judgements: Eliciting Experts' Probabilities.*Chichester, UK: John Wiley & Sons Publishing, ix+327 p.
80. Orlov, A.I. (2002). *Expert Evaluations*. Moscow, RF: Institute of Higher Statistical Technologies and Econometrics Press, 31 p.
81. Orlov, A.I. (2004). *NonnumericalStatistics*. Moscow, RF: MZ-Press, 513 p.
82. Ott, R.L., Longnecker, M. (2008). *An Introduction to Statistical Methods and Data Analysi.* Belmont, USA: Brooks / Cole, Cengage Learning, 1273 p.
83. Podinovski, V.V. (2007). *Introduction to the Theory of Criteria' Importance in Multi-Criteria Problems of Decision-Making*. Moscow, RF: FizMatLit, 64 p.
84. Podinovski, V.V., Potapov, M.A. (2013). Weighted sum method in the analysis of multi-criteria decisions: pro et contra. *Scientific Journal of Business Informatics,* 3(25), 41-48.
85. Puzikov, O.S. (2000). *Course of Lectures on Socio-Economic Forecasting.* Rostov-on-Don, RF: Don State Technical University Press, 124 p.
86. Raiffa, H. (1953). Arbitration schemes for generalized two-person games, *In: "Contributions to the Theory of Games II"* (Ed. by H.W.Kuhn and A.W.Tucker). Annals of Mathematics Studies 28, Princeton, USA: Princeton University Press, 361-387.
87. Ramm, A.G. (2005). *Inverse Problems: Mathematical and Analytical Techniques with Applications to Engineering.* Boston, USA: Springer Science Publishing, 442 p.
88. Rao, C.R. (2002). *Linear Statistical Inference and its Applications.* New York, USA: John Wiley & Sons Publishing, 625 p.
89. Rao, M. (2005). *Knowledge Management Tools and Techniques: Practitioners and Experts Evaluate KM Solutions*. Amsterdam, The Netherlands, Elsevier Publishing, 438 p.
90. Redies, H.A. (Ed.) (1974). *Parameter estimation techniques and applications in aircraft flight testing*. Washington, USA: National Aeronautics and Space Administration (NASA) Press, 385 p.
91. Roth, A.E. (1977). Independence of irrelevant alternatives, and solutions to Nash's bargaining problem. *Journal of Economic Theory, 16*(2), 247-251.
92. Rowell, D. (2017). *Analysis and Design of Feedback Control Systems*. Boston, USA: MIT Open Course Ware, Available on-line at: https://ocw.mit.edu/index.htm [Accessed: November 11, 2017].
93. Rowell, D. (2017). *Dynamic Systems and Control.* Boston, USA: MIT OpenCourseWare, Massachusetts Institute of Technology, Available on-line at: https://ocw.mit.edu/index.htm [Accessed: November 11, 2017].

94. Rowell, D.,Wormley, D.N. (1997). *System Dynamics: An Introduction*. Upper Saddle River, USA: Prentice Hall Publishing, 592 p.
95. Saaty, T.L. (2000). *Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process*. Pittsburgh, USA: RWS Publishing, 463 p.
96. Sawaragi, Y., Nakayama, H., Tanino, T. (1985). *Theory of Multiobjective Optimization*, Orlando, USA: Academic Press, 296 p.
97. Schreiber, G., McCaul, R.M., Anjewierden, A., de Hoog, R., Shadbolt, N.R., van de Velde, W., Wielinga, B.J. (1999). *Knowledge Engineering and Management: The CommonKADS Methodology*. Boston, USA: MIT Press, 455 p.
98. Shemilt, I., Mugford, M., Vale, L., Marsh, K., Donaldson, C. (2010). *Evidence-Based Decisions and Economics: Helath Care, Social Welfare, Education and Criminal Justice*. Chichester, UK: John Wiley & Sons Publishing, 206 p.
99. Sizikov, V.S. (2003). On methods of residual in the solution of ill-posed problems, *Journal of Computational Mathematics and Mathematical Physics, 23*(9), 1294-1312.
100. Steuer, R.E. (1986). *Multiple Criteria Optimization: Theory, Computations, and Application.* New York, USA: John Wiley & Sons Publishing, 586 p.
101. Tikhonov, A.N., Arsenin, V.Ya. (1977). *Solutions of Ill-Posed Problems*. New York, USA: V.H.Winston& Sons Publishing, 258 p.
102. Turin, Y.N., Litvak, B.G., Orlov, A.I., Satarov, G.A., Shmerling, D.S. (1981). *Analysis of Non-Numerical Information*. Moscow, RF: Scientific Council of USSR Academy of Sciences on the Complex Problems Press, 80 p.
103. Turin, Y.N., Vasilevich, A.P., Andrukovich, P.F. (1977). Statistical models of ranking, *In the book: "Statistical methods of expert evaluation analysis",* Moscow, RF: Science Publishing, 30-58.
104. Van der Linden, W.J., Hambleton, R.K. (1997). *Handbook of Modern Item Response Theory*. New York, USA: Springer-Verlag Publishing, 510 p.
105. Various economic, financial, social, political, legal, educational, scientific innovative, ecological, cultural, etc. factors,Available on-line at: http://www.worldbank.org/, http://www.eiu.com/home.aspx, https://www.weforum.org/, http://www.transparency.org/, https://wiiw.ac.at/, https://www.credit-suisse.com, http://www.imf.org/external/index.htm, http://ec.europa.eu/eurostat/data/database, http://www.heritage.org/, https://www.ihs.com/industry/economics-country-risk.html http://www.prosperity.com/, https://www.numbeo.com, http://hdr.undp.org/en, http://bnb.bg/ [Accessed: November 11, 2017].
106. Vasiliev, F.P. (1974). *Lectures on Methods for Solution of Extremum Problems.* Moscow, USSR: Lomonosov Moscow State University Press, 376 p.
107. Ventsel, Ye.S. (1988). *Operations Research: Problems, Principles, Methodology*. Moscow, RF: Science Publishing, 208 p.
108. Verlan, A.F., Sizikov, V.S. (1986). *Integral Equations: Methods, Algorithms.* Computer Program, Kyiv, Naukova Dumka Publishing, 544 p.
109. Vincke, P. (1992). *Multi-criteria Decision-Aid*. Chichester, UK: John Wiley & Sons Publishing, 174 p.
110. Vizureanu, P. (2010). *Expert Systems.* Vukovar, Croatia: Intech Publishing, 238 p.
111. Yoshida, K. (1995). *Functional Analysis*. Berlin, Germany: Springer-Verlag, 501 p.
112. Yunusov, S.M., Guseynov, Sh.E., Andreyev, S.A., Labendik, V.P. (2013). Application of Tikhonov's Regularization Method for Stable Finding the Diagnostic Matrix of GTE with a priori Given Accuracy, Part I: Features of Diagnosing and Development of Mathematical Model, *Journal of Mathematical Applications in Science & Mechanics*, 14, 109-114.

113. Yunusov, S.M., Guseynov, Sh.E. (2011). New approach to the formation of the adequate diagnostic matrix of the gas turbine engine. *In: Proceedings of the 25th European Conference on Modelling and Simulation*, Krakow, Poland, 362-369.
114. Yunusov, S.M., Labendik, V.P., Guseynov, Sh.E. (2014). *Monitoring and Diagnostics of Aircraft Gas Turbine Engines: Improvement of Models and Methods for Diagnosis of Gas Path of Gas Turbine Engines*. Saarbrucken, Germany: LAP Publishing, 204 p.
115. Zaikin, P.N. (1978). Systems for complete mathematical processing of the results of spectrometric experiments, Dissertation to obtaining the scientific degree Doctor of Science in Mathematics & Physics,Dubna, RF: Joint Institute for Nuclear Research Press, 224 p.